

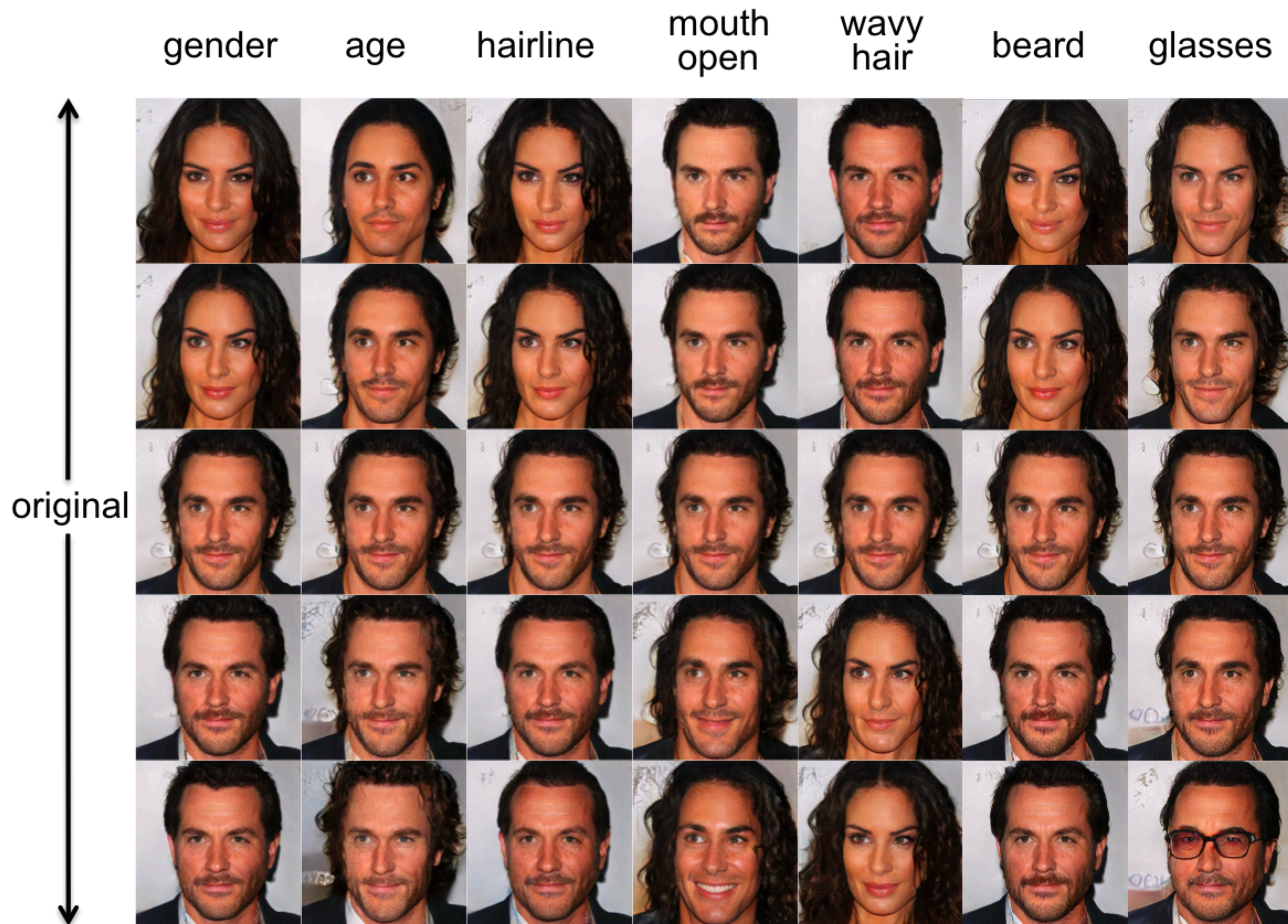
# Evaluating the Disentanglement of Deep Generative Models with Manifold Topology

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Andrew Ng, Gunnar Carlsson, Stefano Ermon

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ICLR 2021

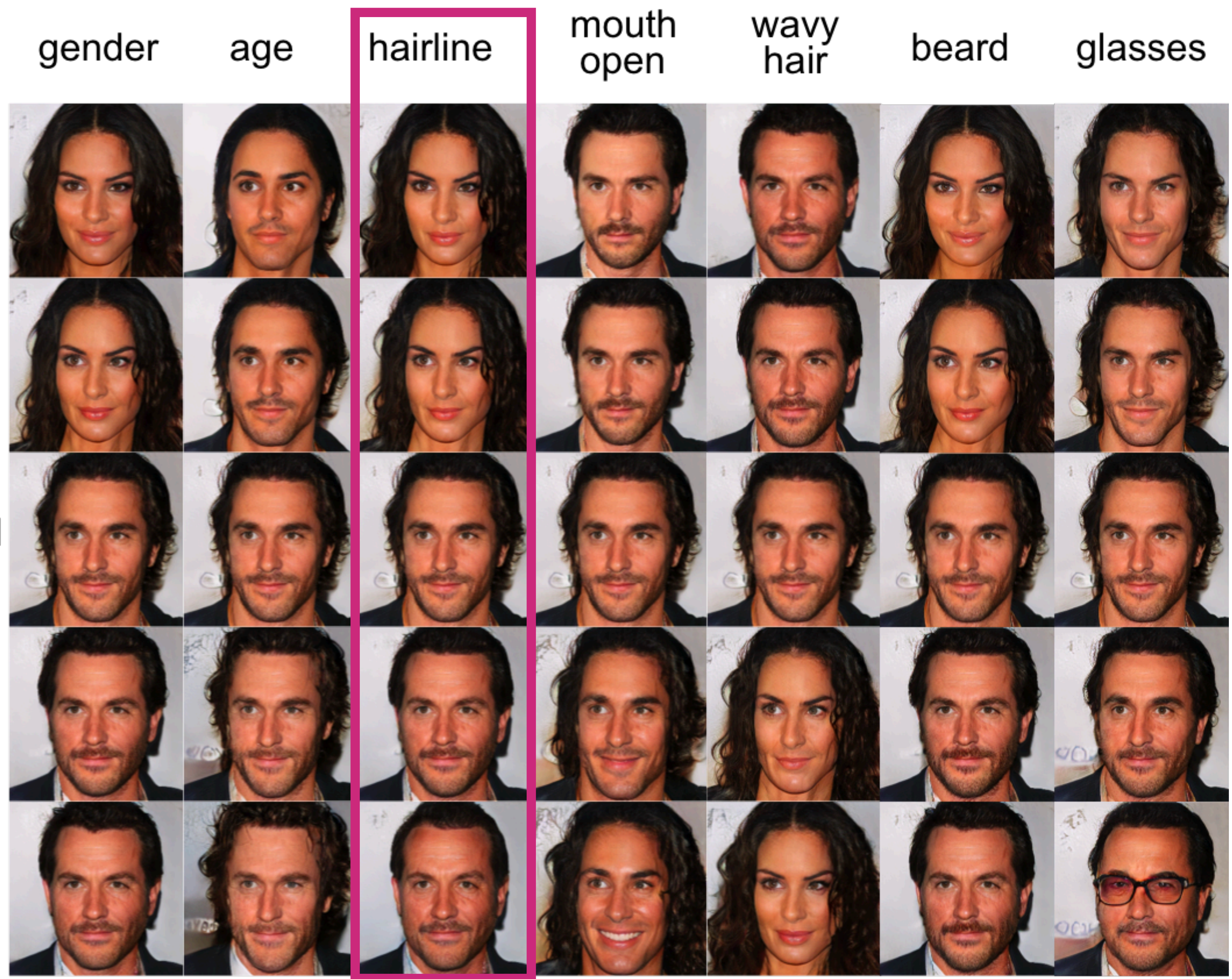




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original



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d170b1b59255](https://blog.insightdata science.com/generating-custom-photo-realistic-faces-using-ai-d170b1b59255)



# Good Representations Disentangle the Explanatory Factors of Variation\*



Glasses

Beard

Hair

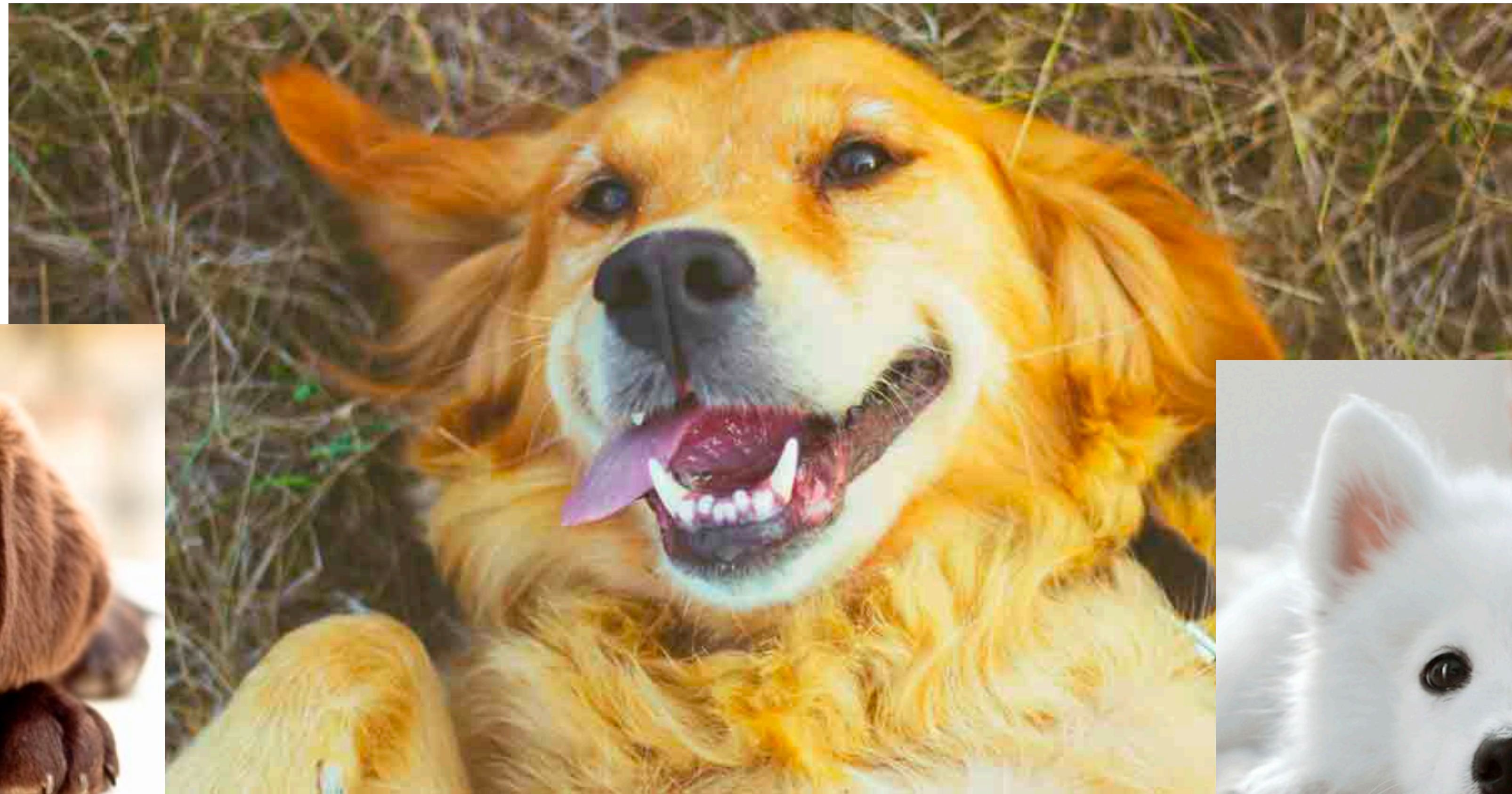
Eyes

Age

\* Y. Bengio, A. Courville, and P. Vincent. Representation Learning: A Review and New Perspectives. *IEEE transactions on pattern analysis and machine intelligence* 35.8 (2013): 1798-1828.



# How would you describe all the major features of *dogs*?



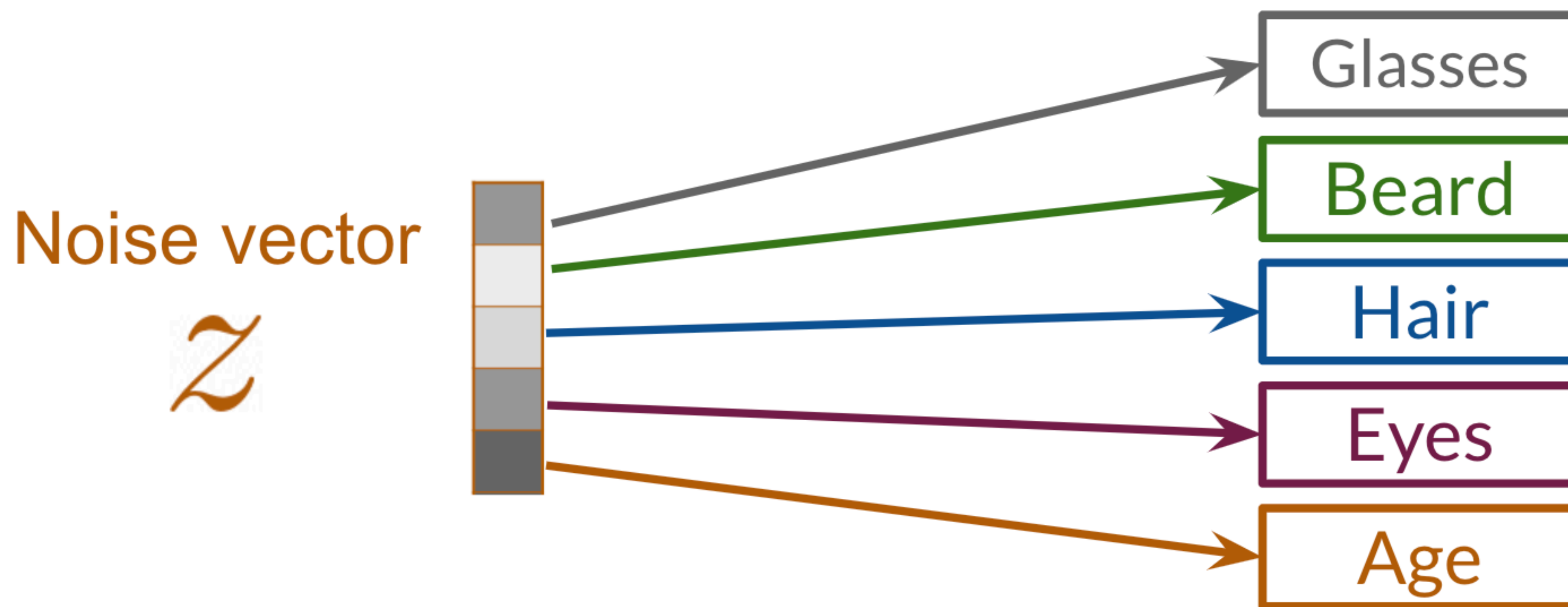


Size  
Fur type  
Nose color  
Ear floppiness  
Tongue droopiness



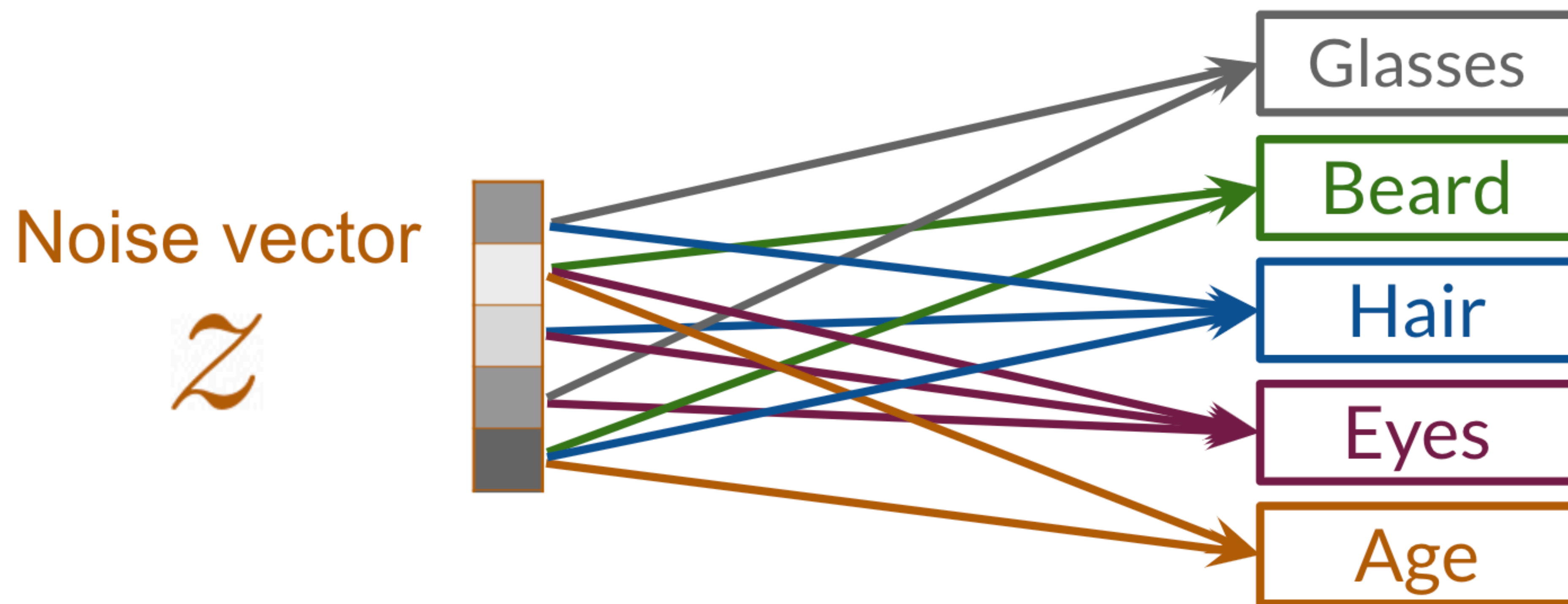


# Z-Space Disentanglement



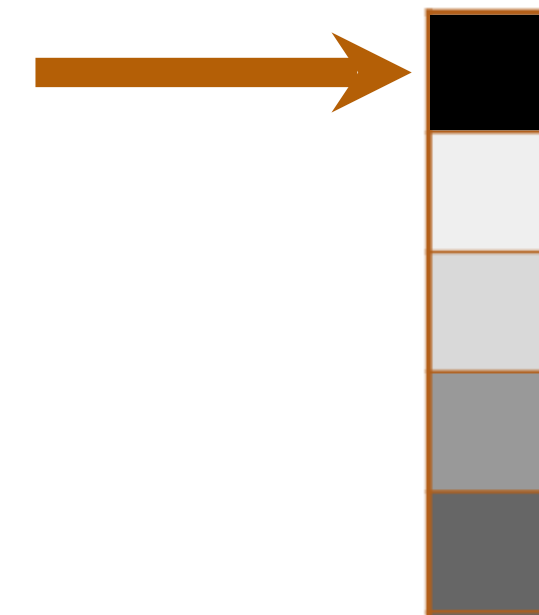
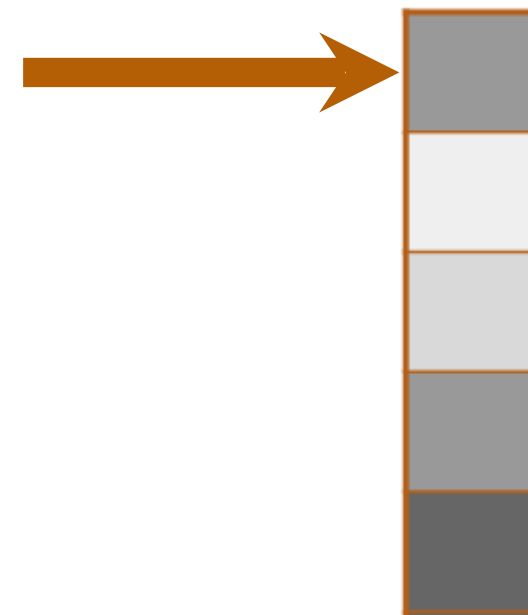


# Z-Space Entanglement



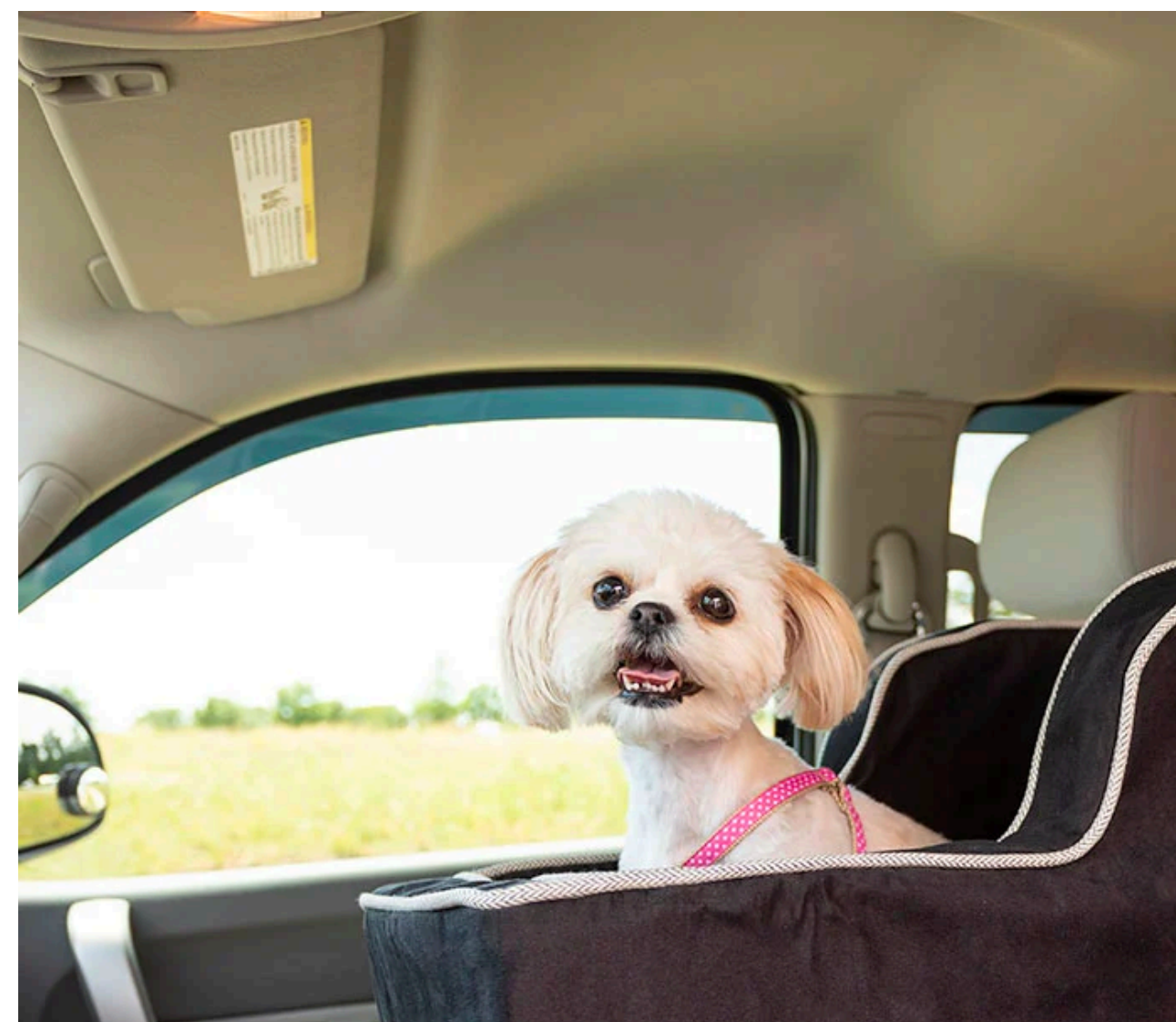


# Disentangled latent dimension



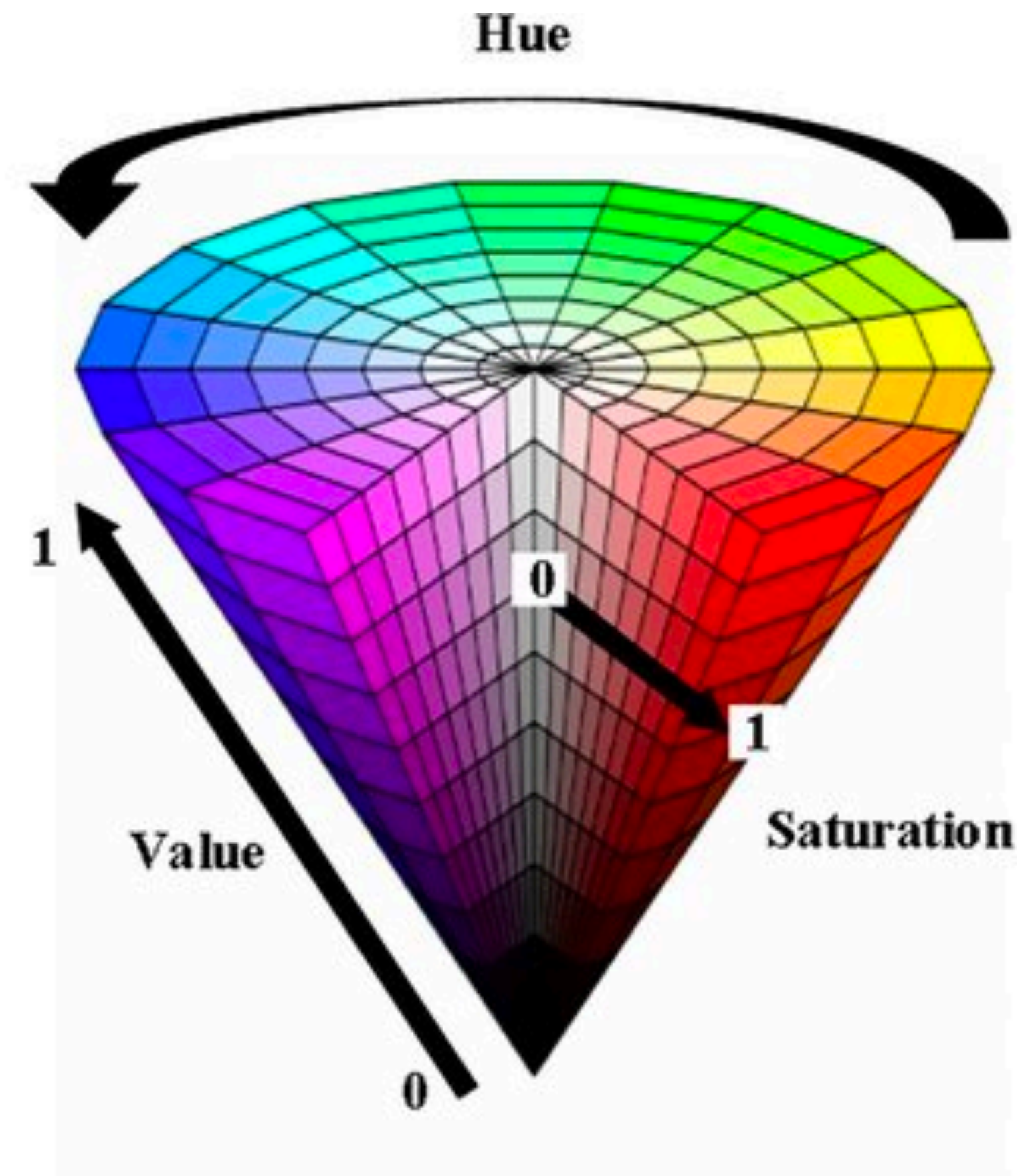
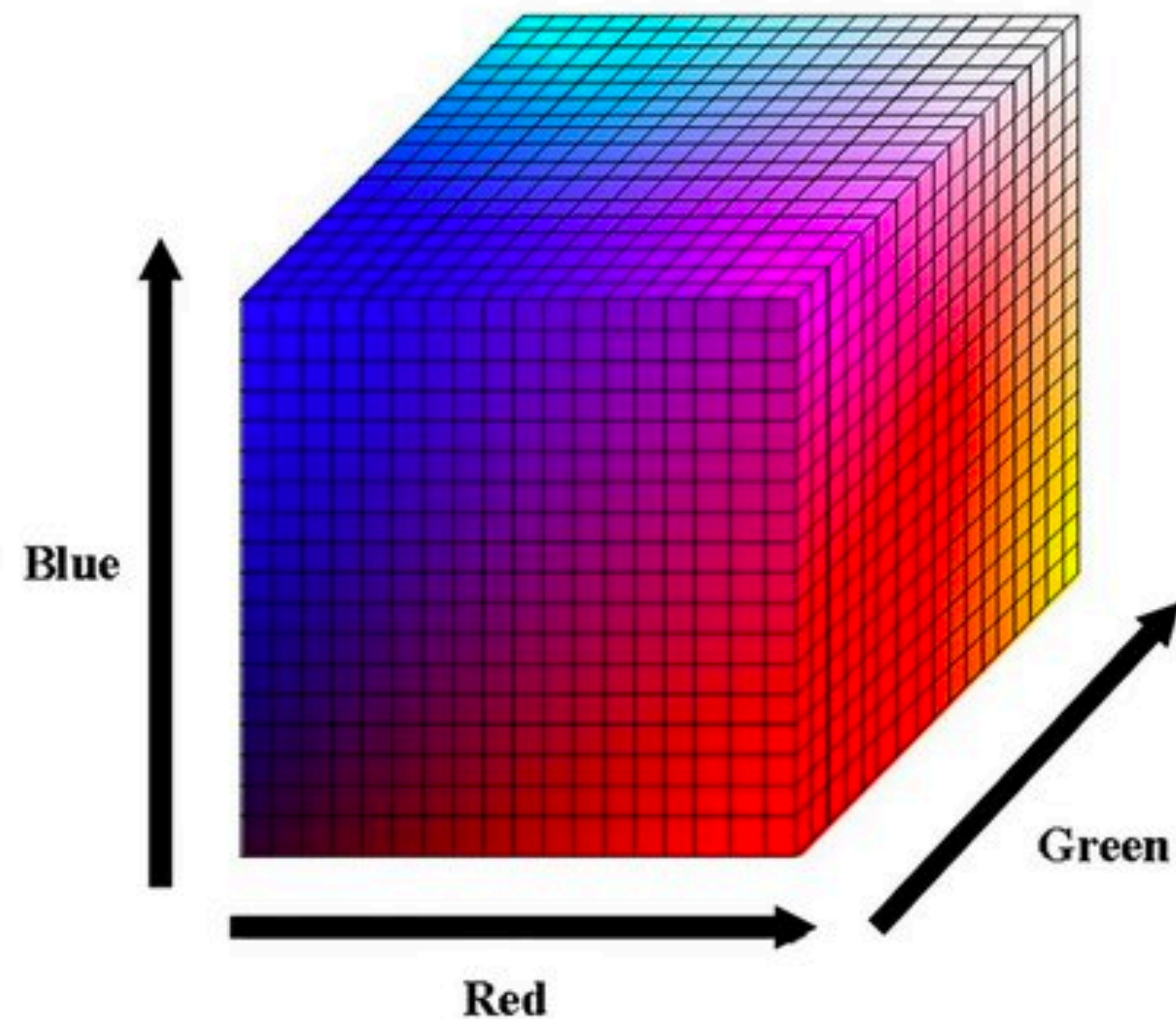


# Entangled latent dimension





# Multiple Factorizations Are Possible



Chen, Rui, Meiling Wang, and Yi Lai. "Analysis of the role and robustness of artificial intelligence in commodity image recognition under deep learning neural network." *Plos one* 15.7 (2020): e0235783.



# *dSprites* Disentanglement Dataset



1. Shape (square, ellipse, heart)
2. Scale (size)
3. Orientation (rotation)
4. X Position
5. Y Position

L. Matthey, I. Higgins, D. Hassabis, and A. Lerchner. *dsprites*: Disentanglement testing sprites dataset. <https://github.com/deepmind/dsprites-dataset/>, 2017.



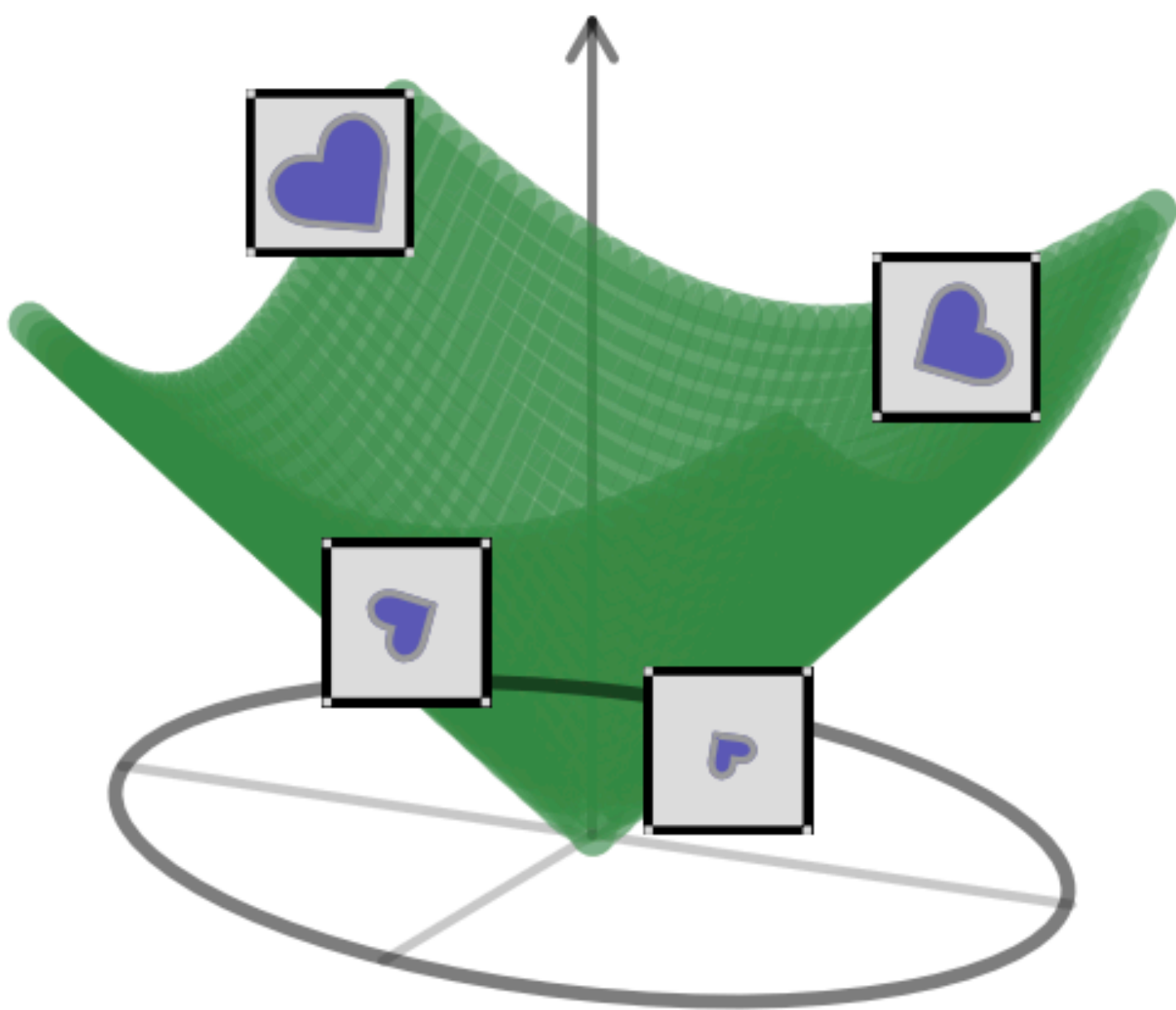
## Prior methods

1. **Supervision** required for a specific factorization
2. Tuned to a **specific dataset**, e.g. custom preprocessing on face images
3. Depends on the **architecture** specifically with an **external model**, e.g. encoder and/or classifier

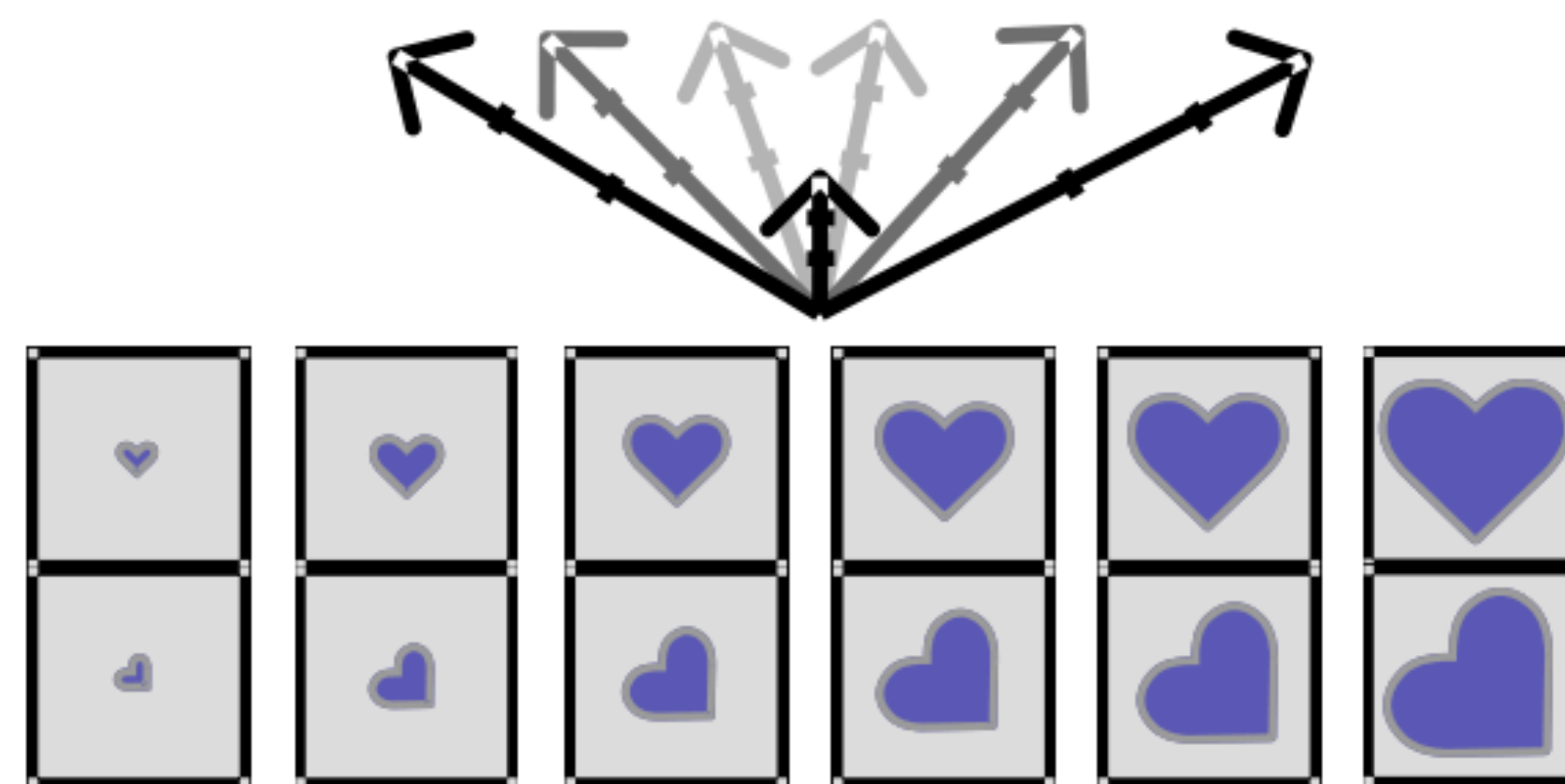
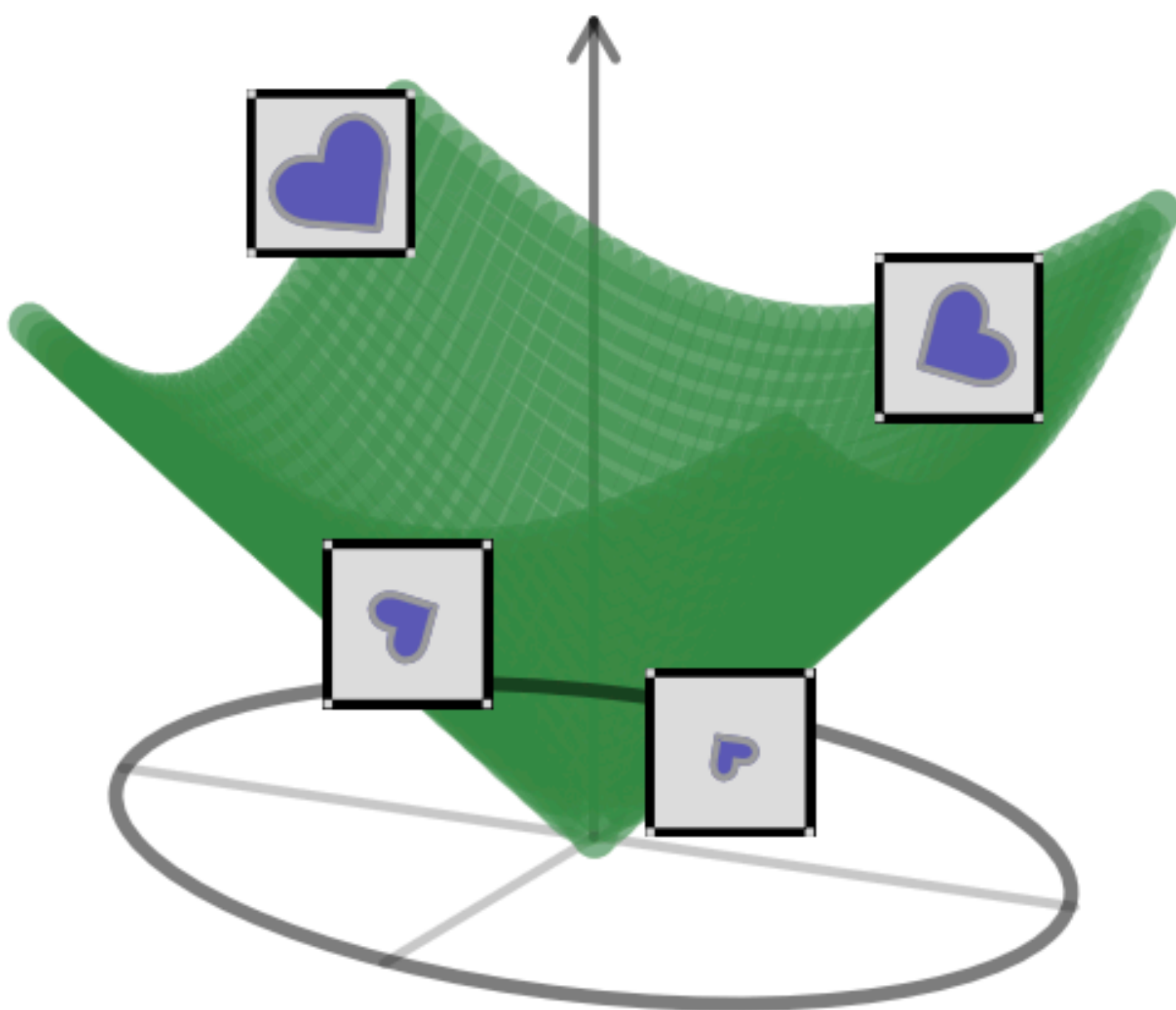
## Ours

1. **Unsupervised** and supervised variants both available
2. Procedure can be **applied across datasets** — and architectures, as above
3. Uses an **intrinsic property** of a generative model, without reliance on external models or custom architectures

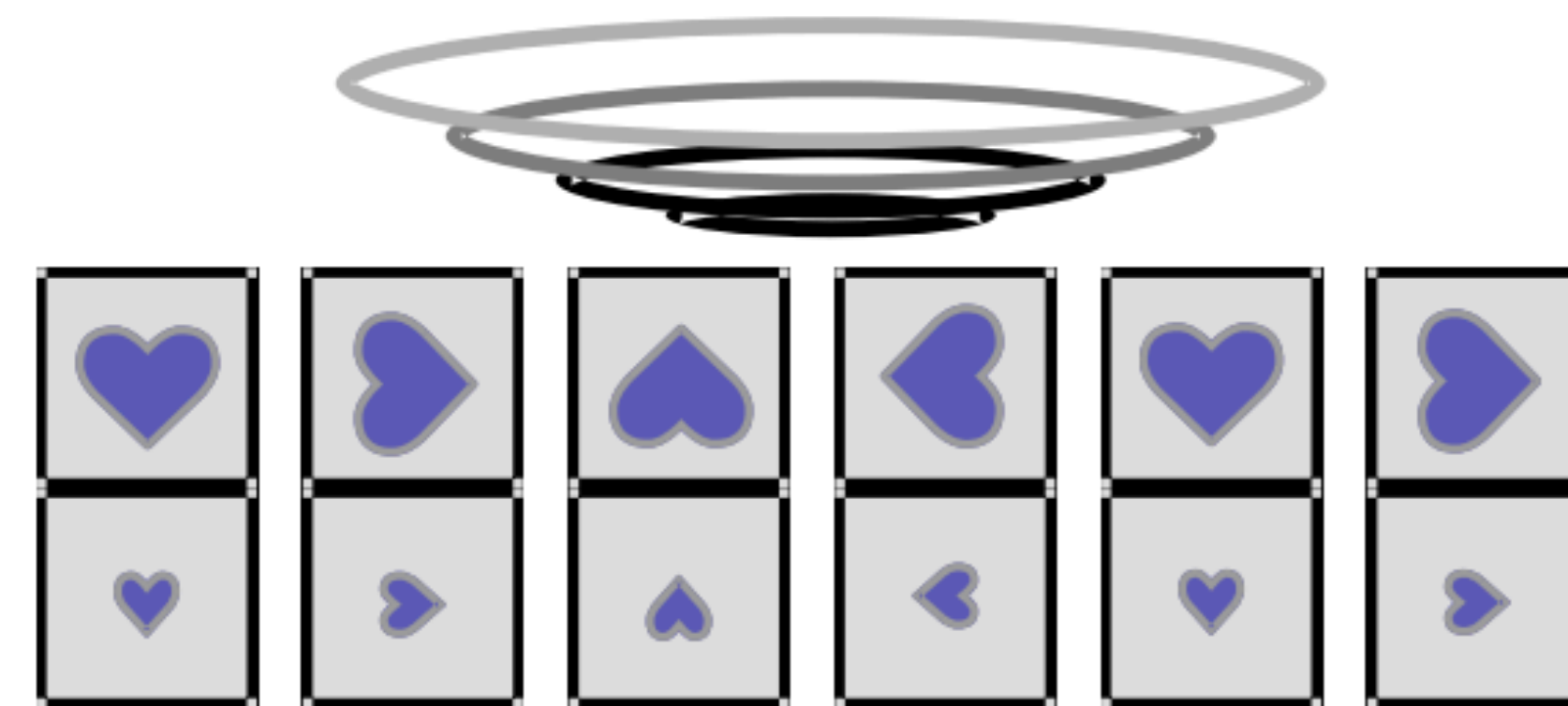






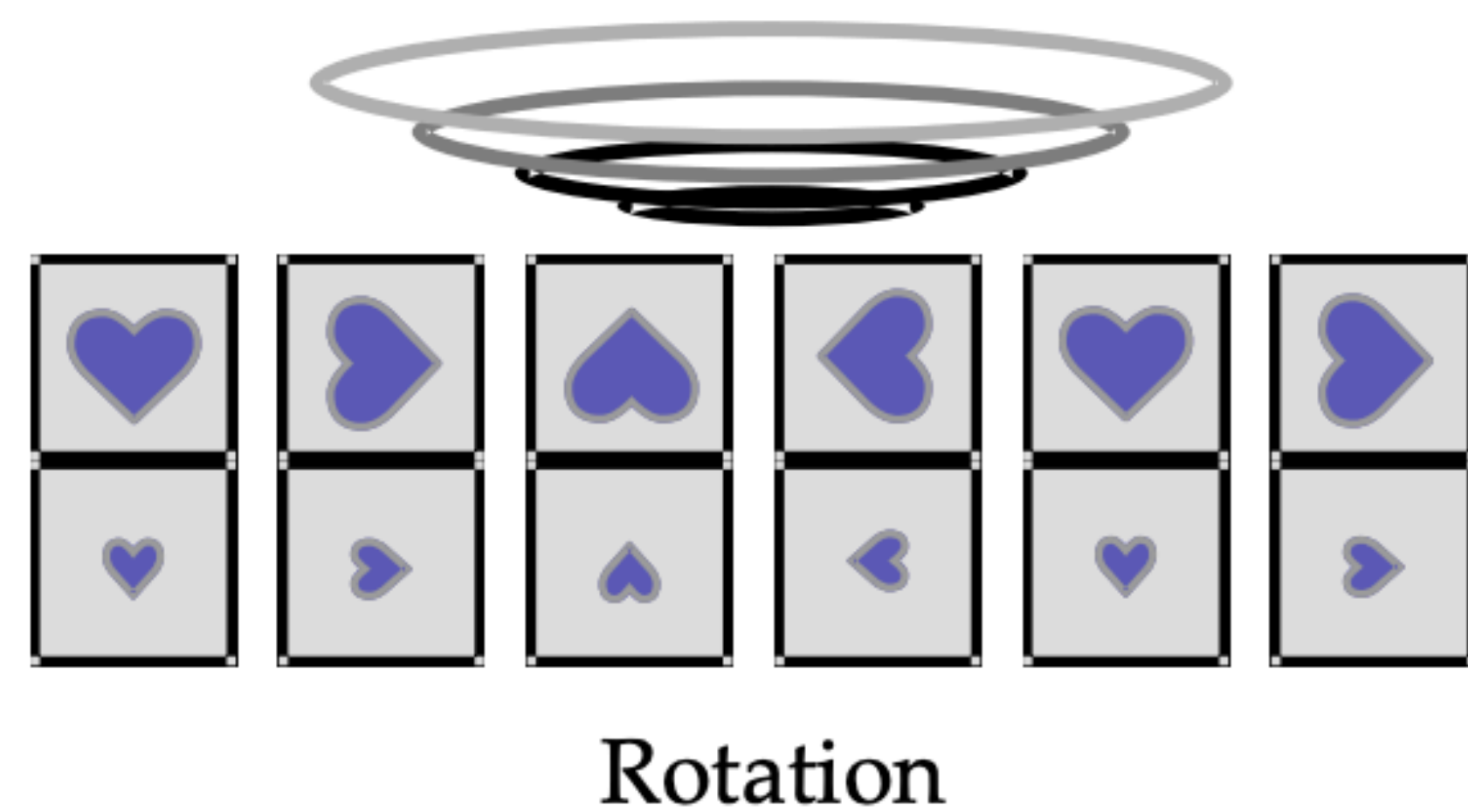
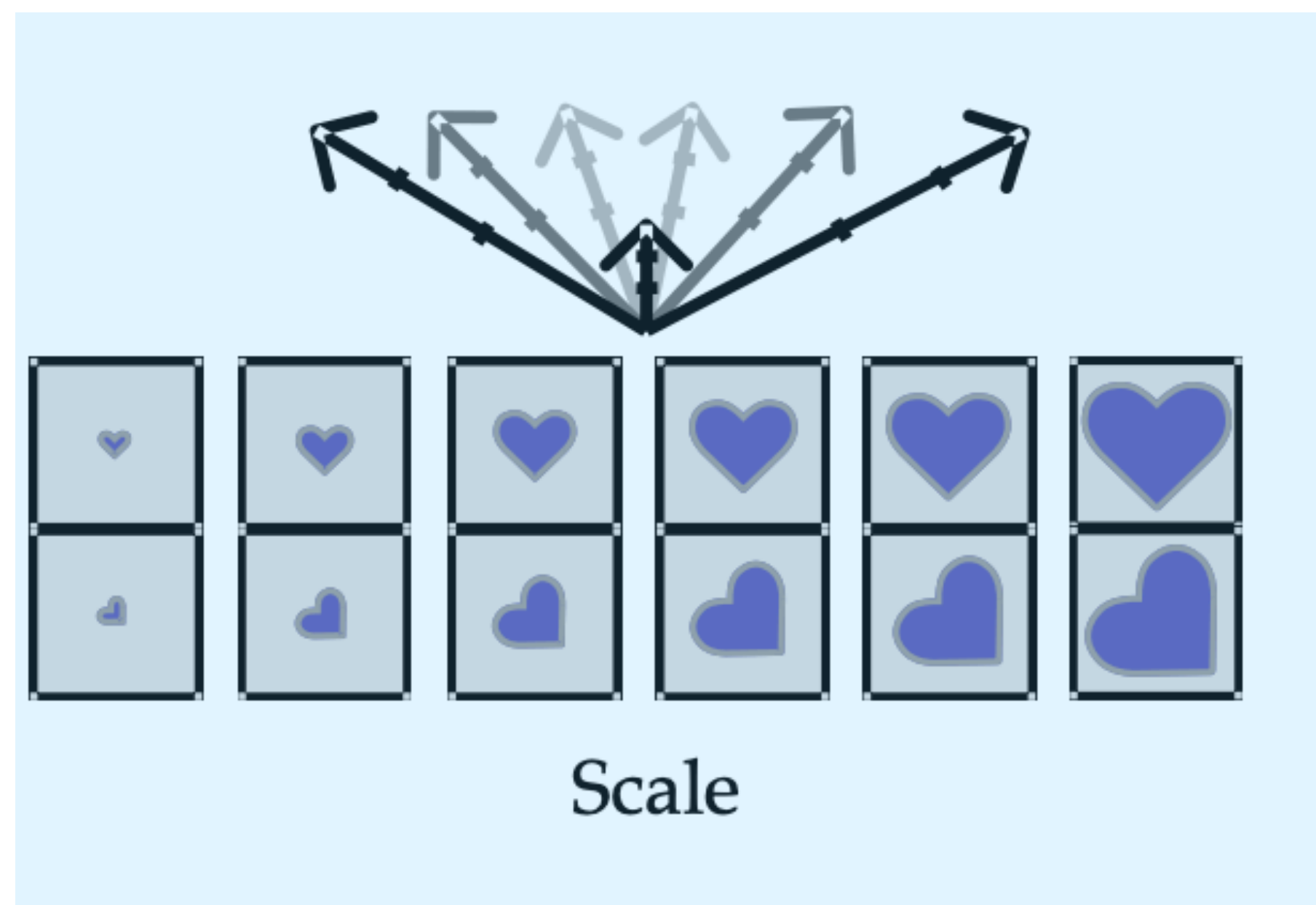
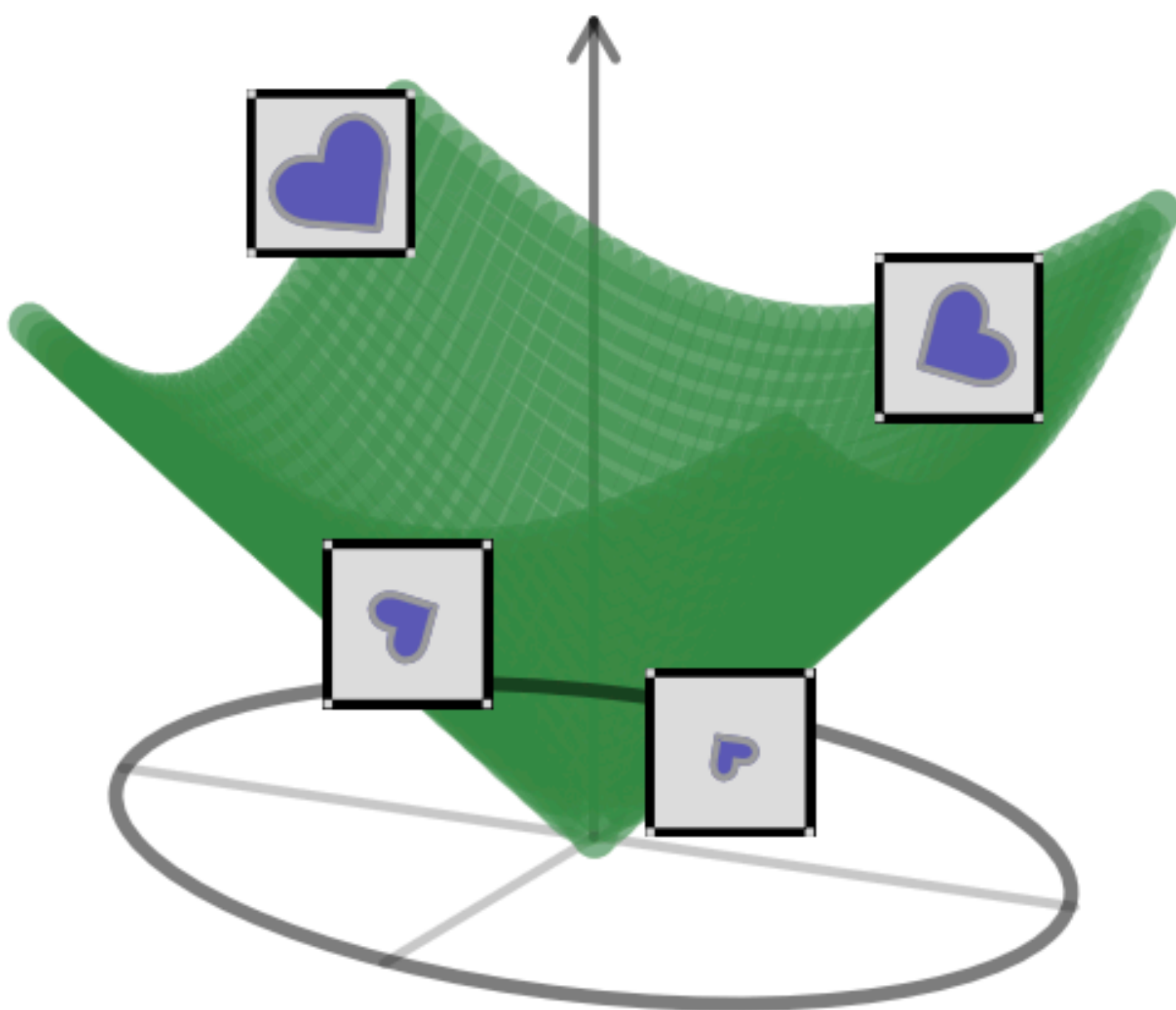


Scale

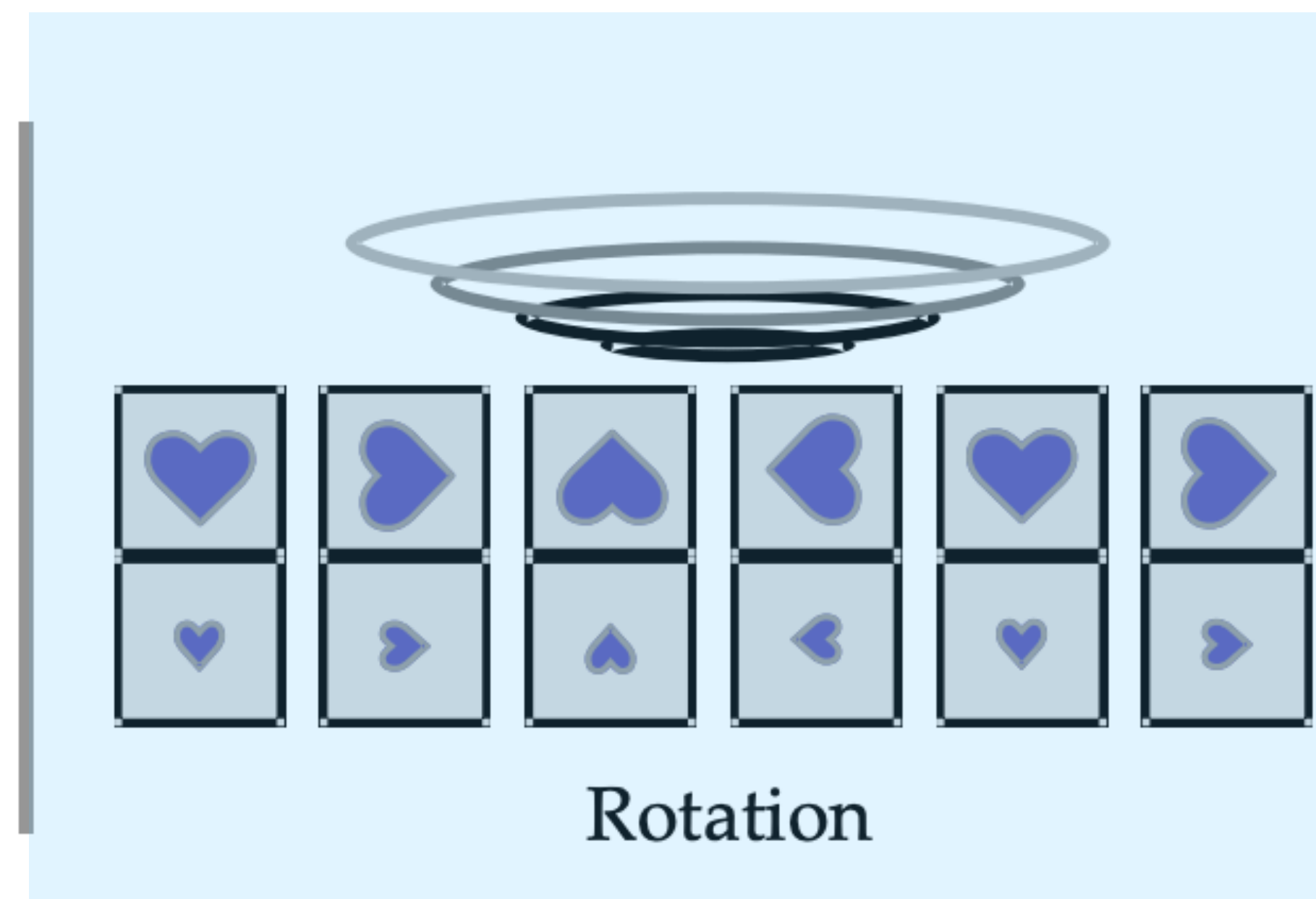
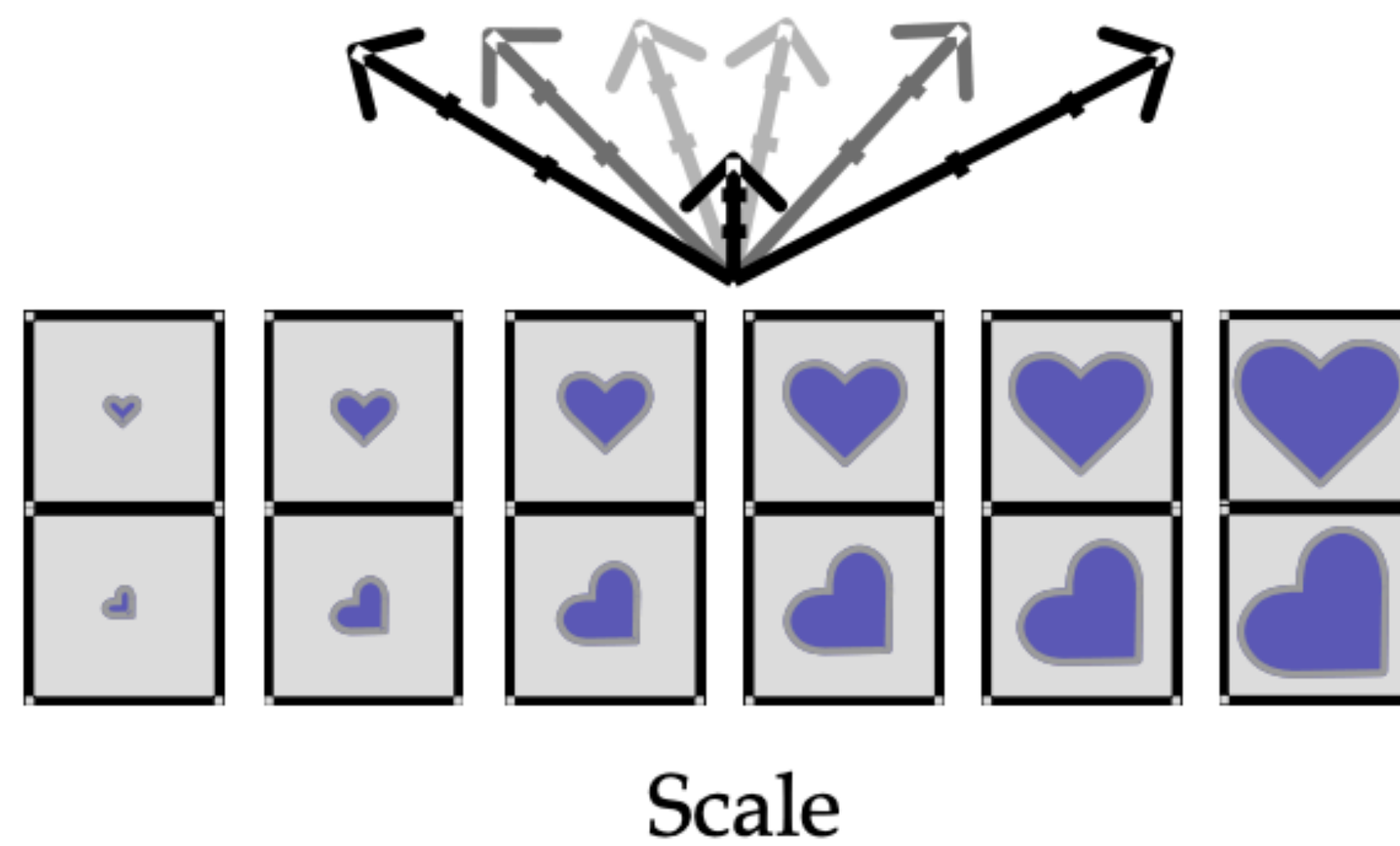
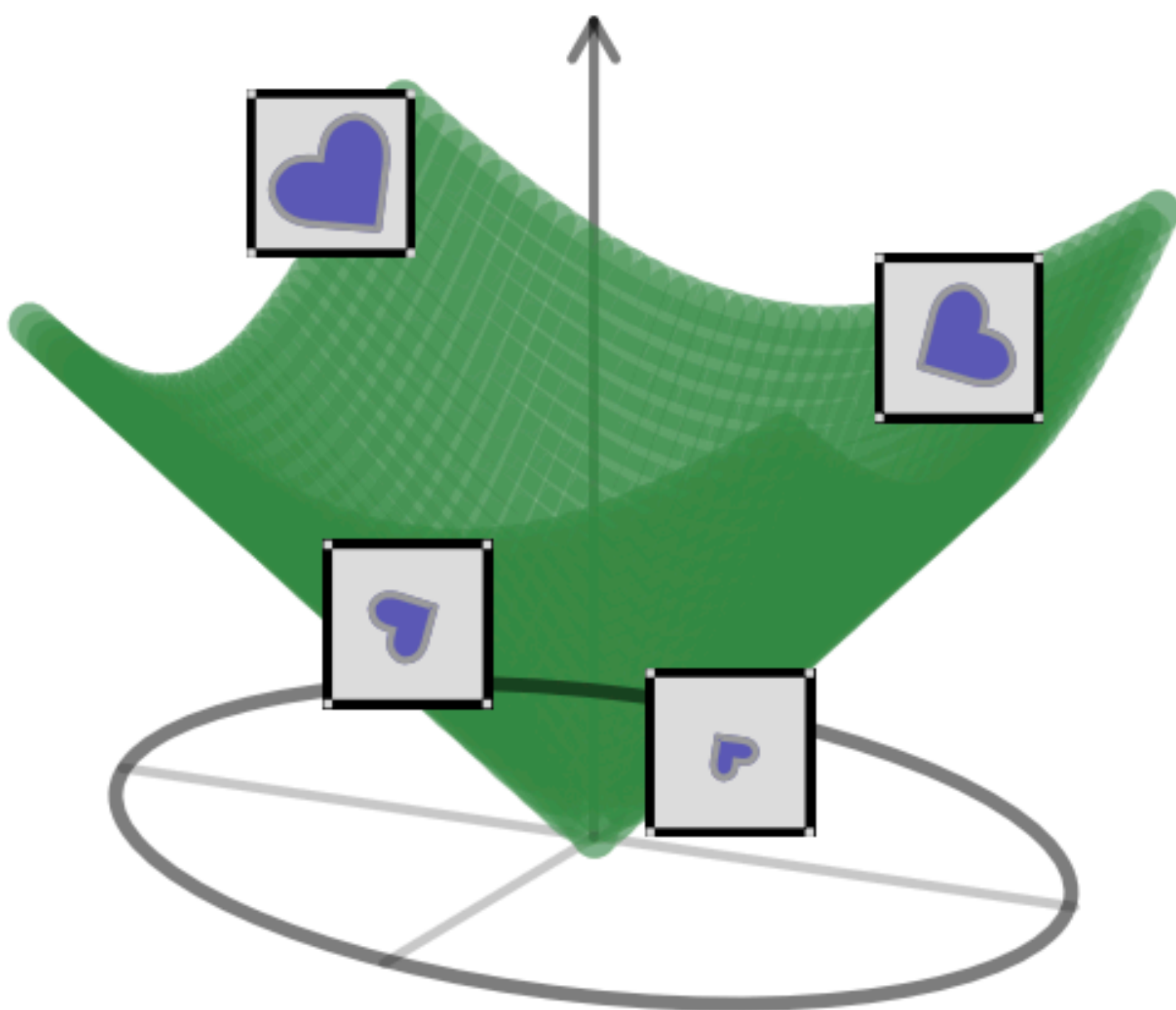


Rotation

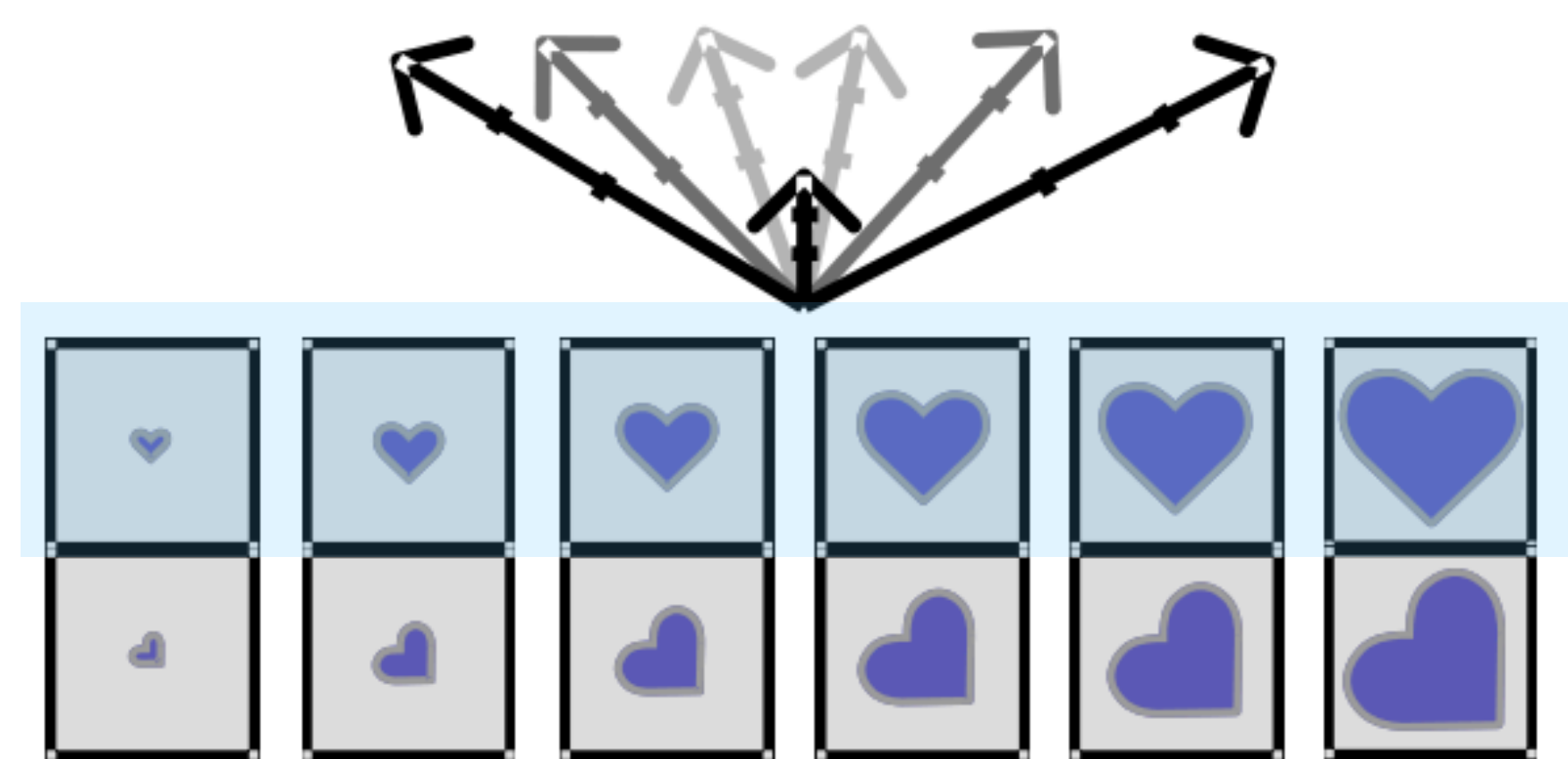
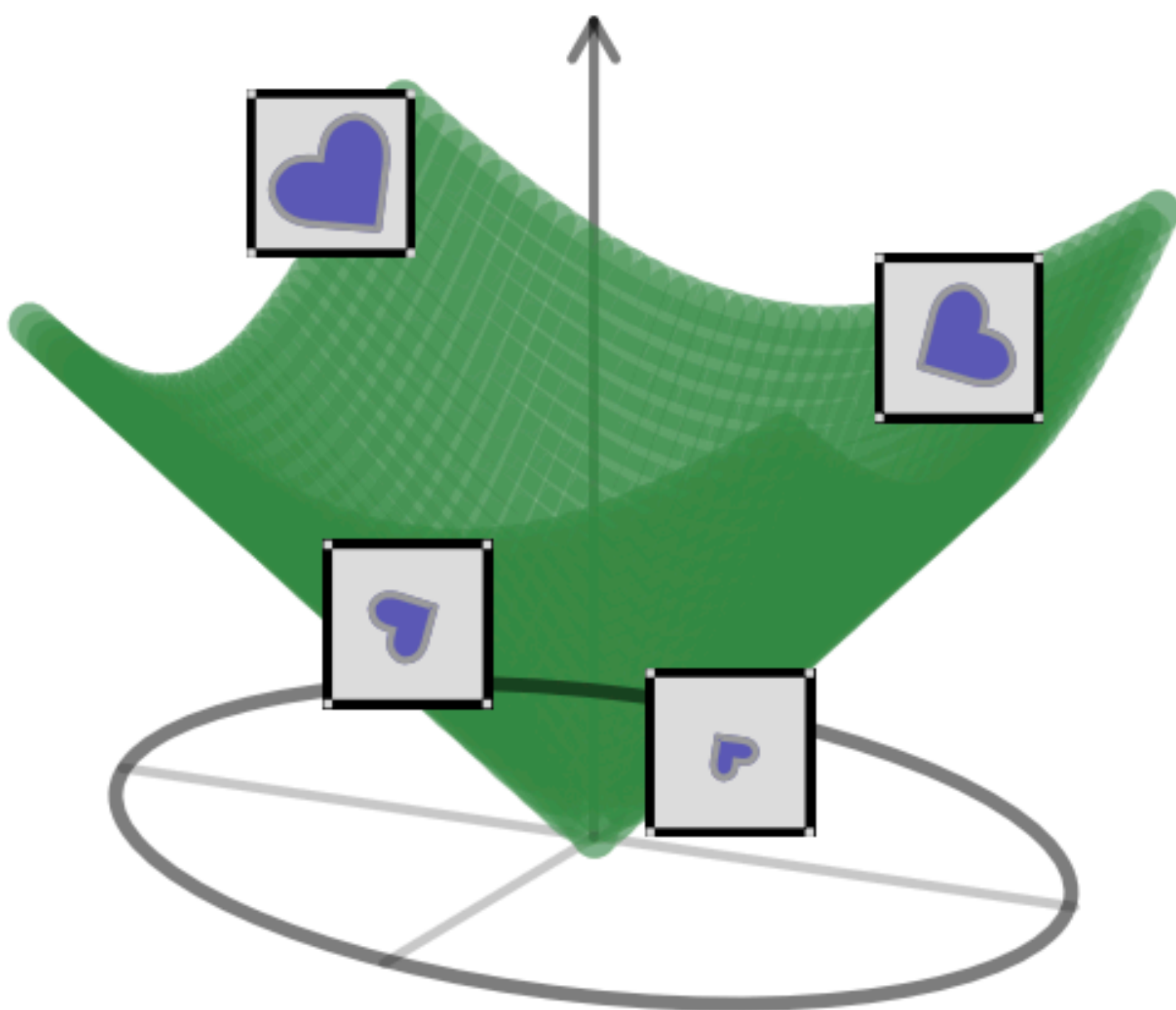




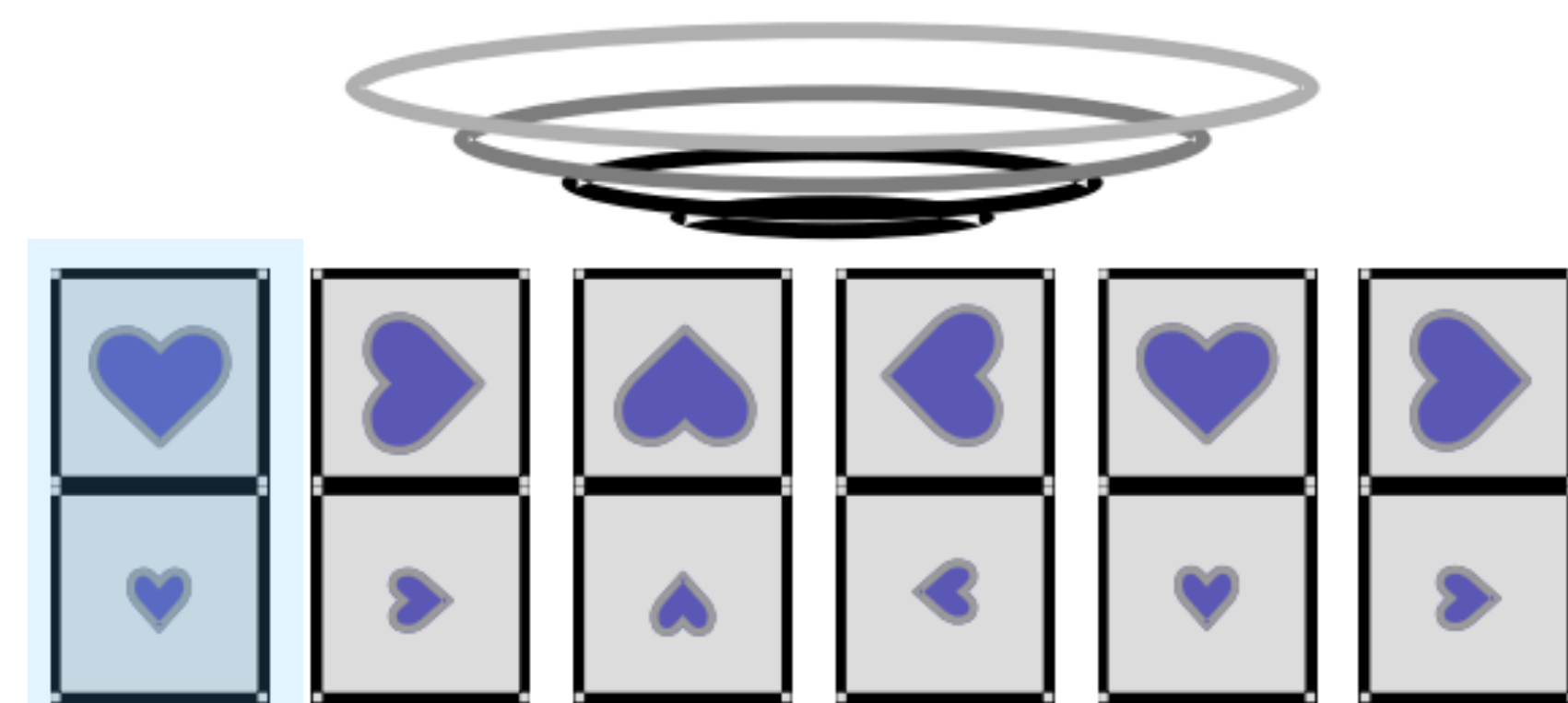






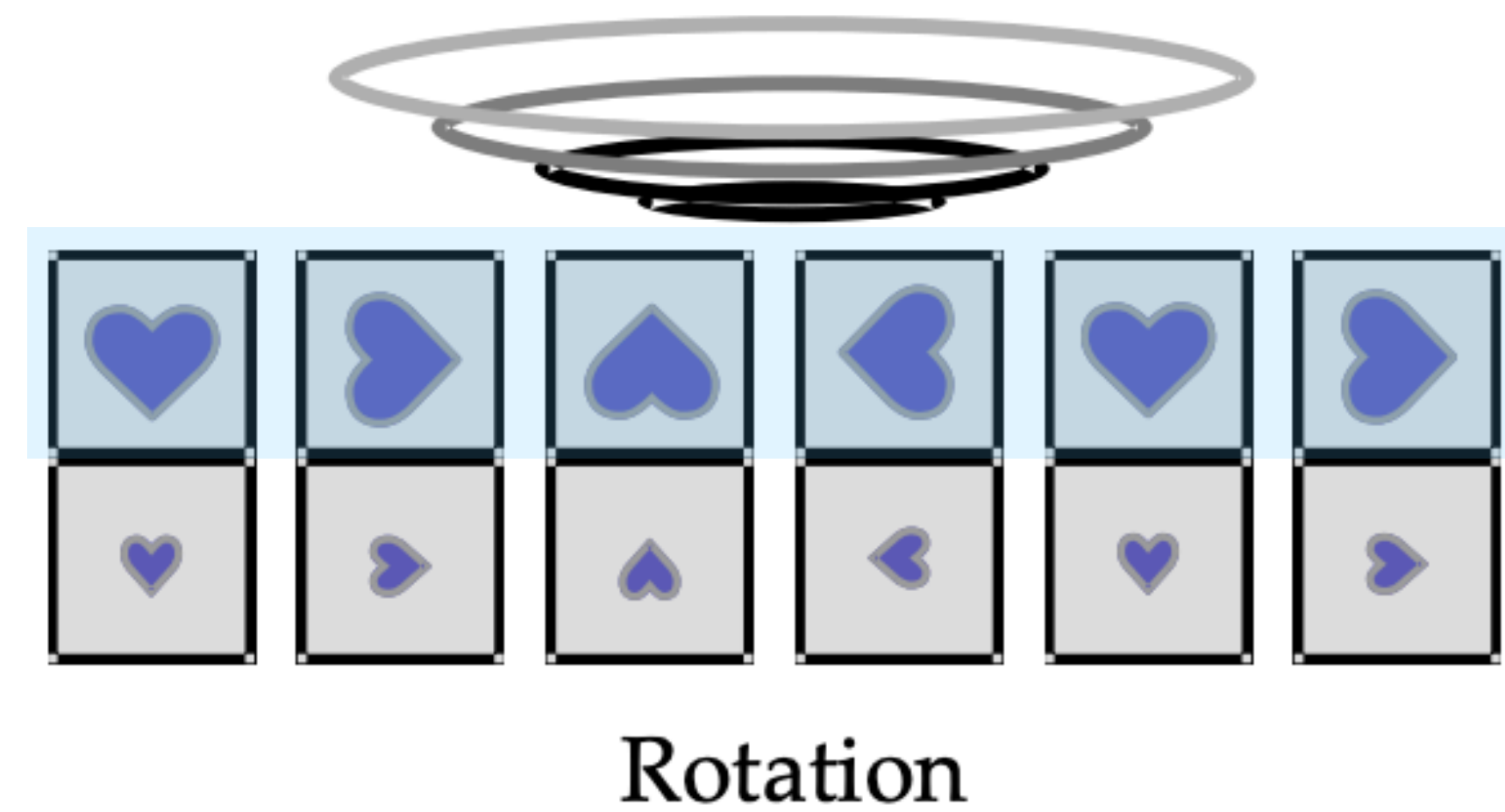
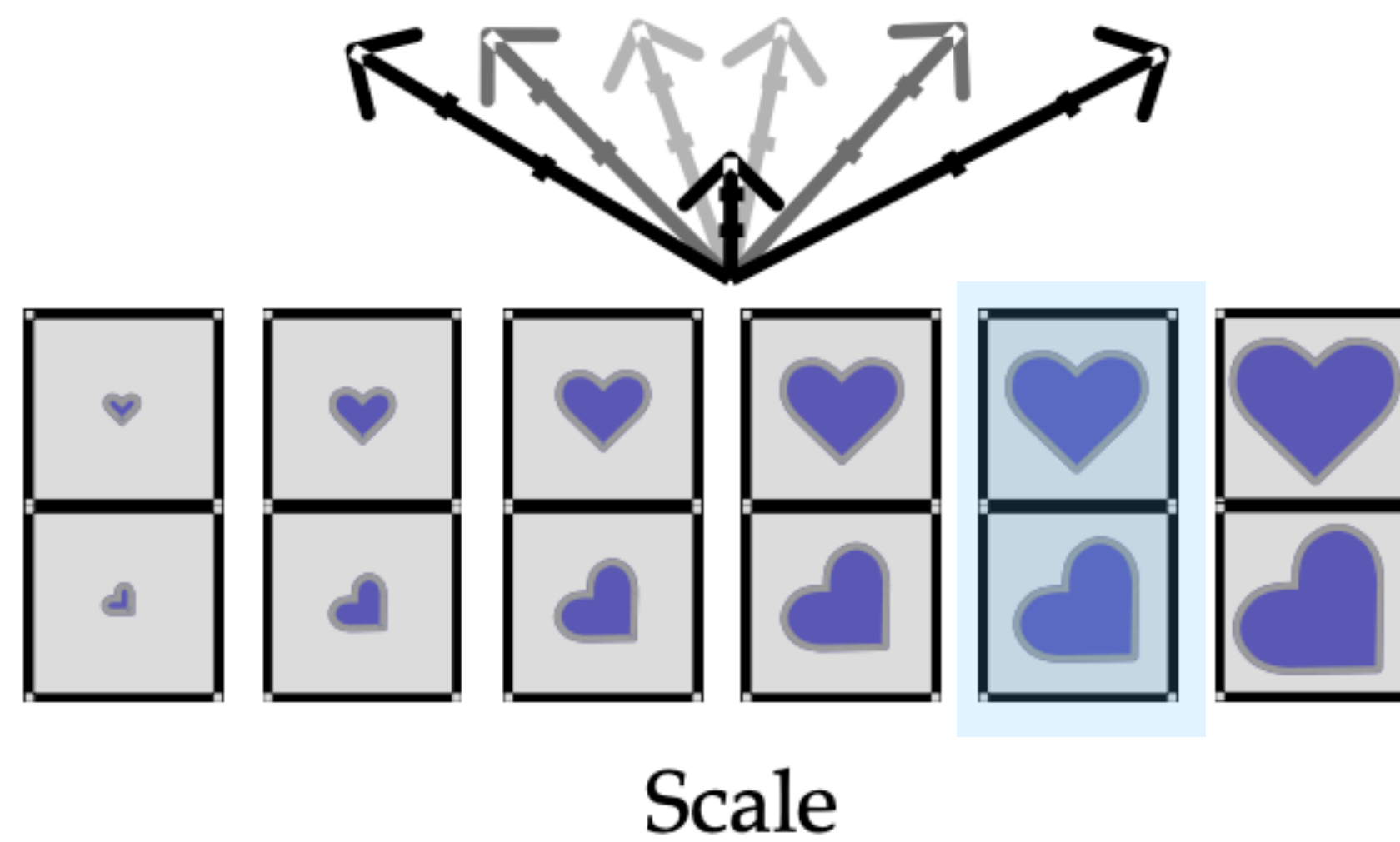
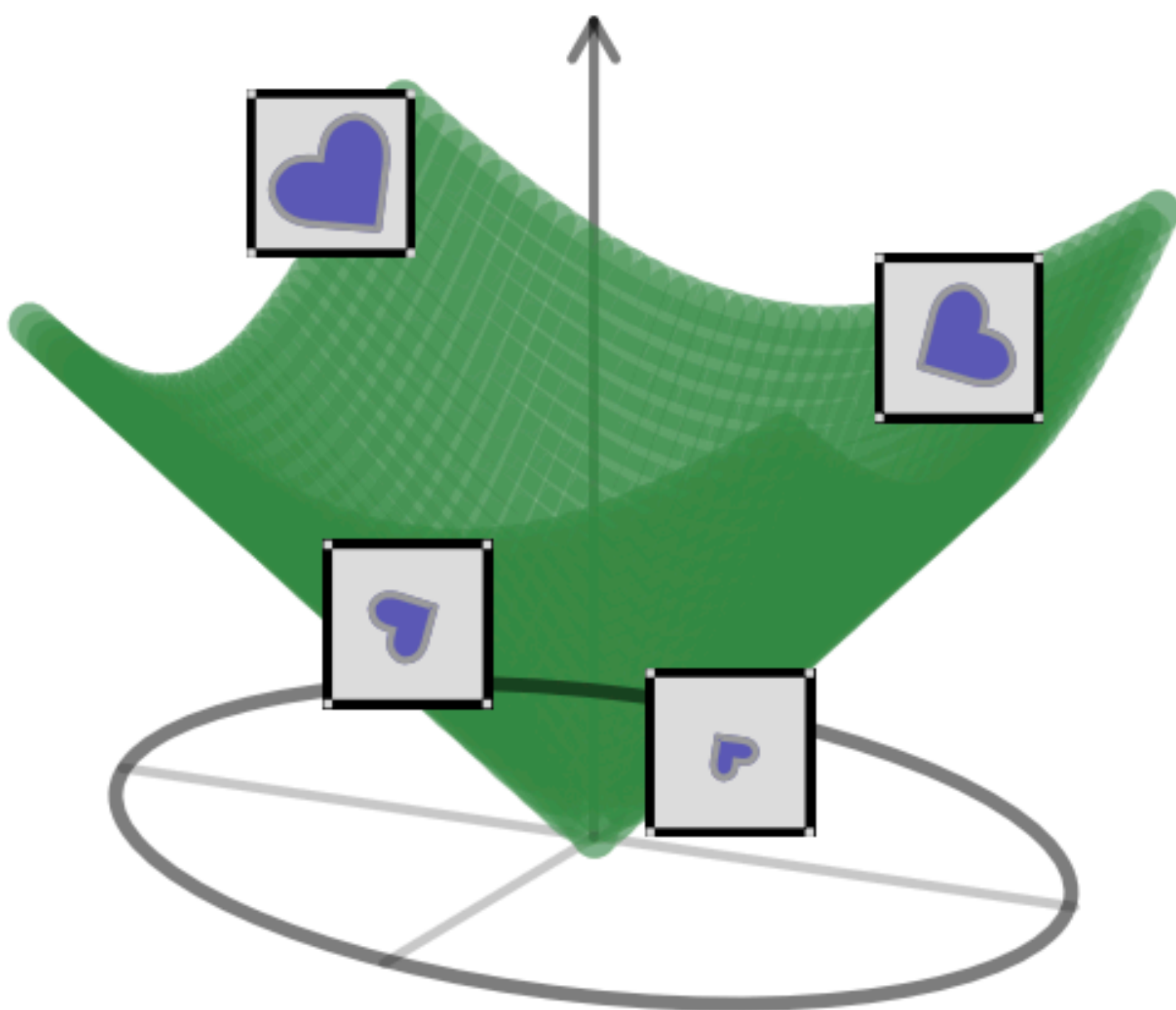


Scale

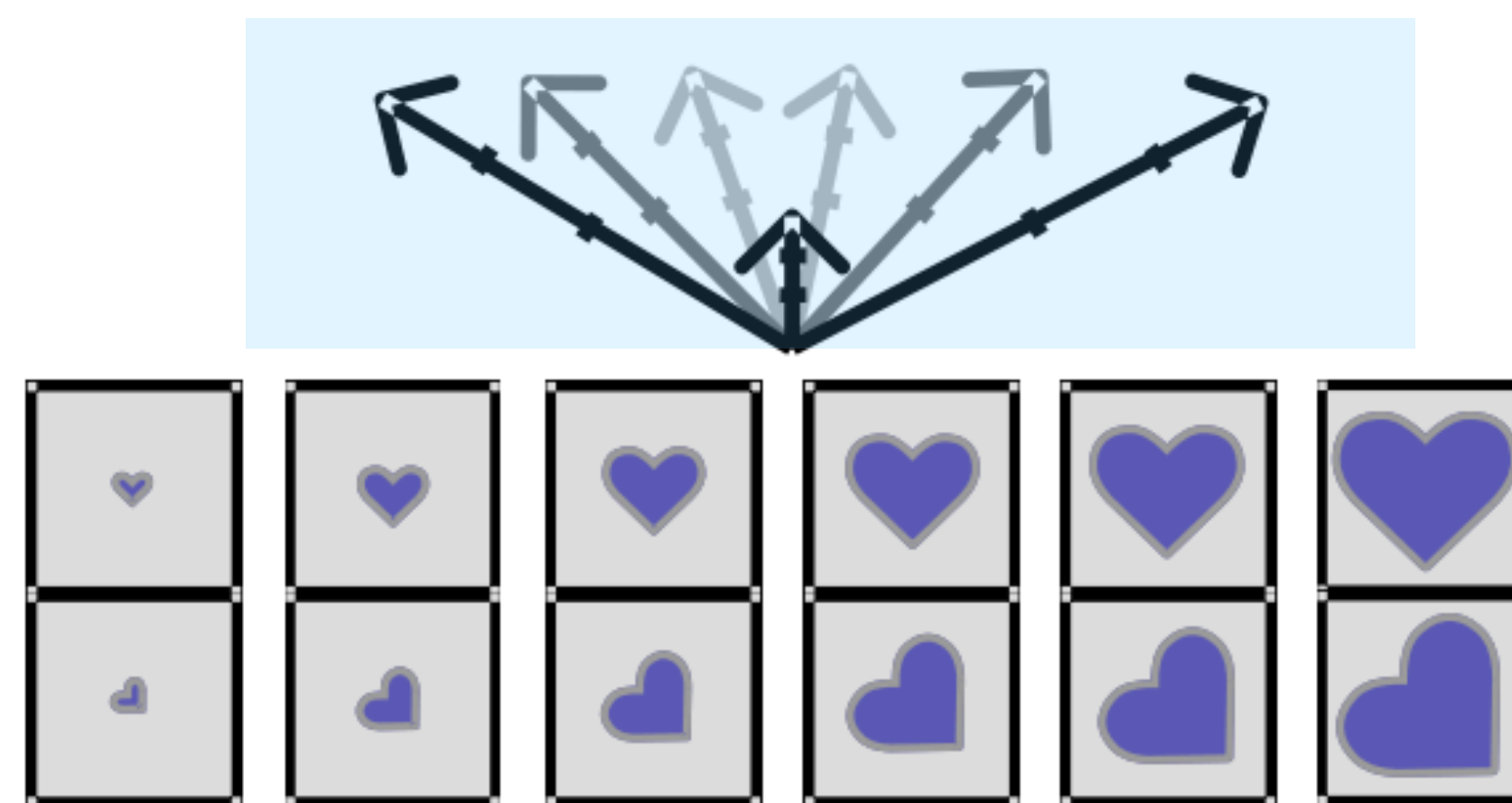
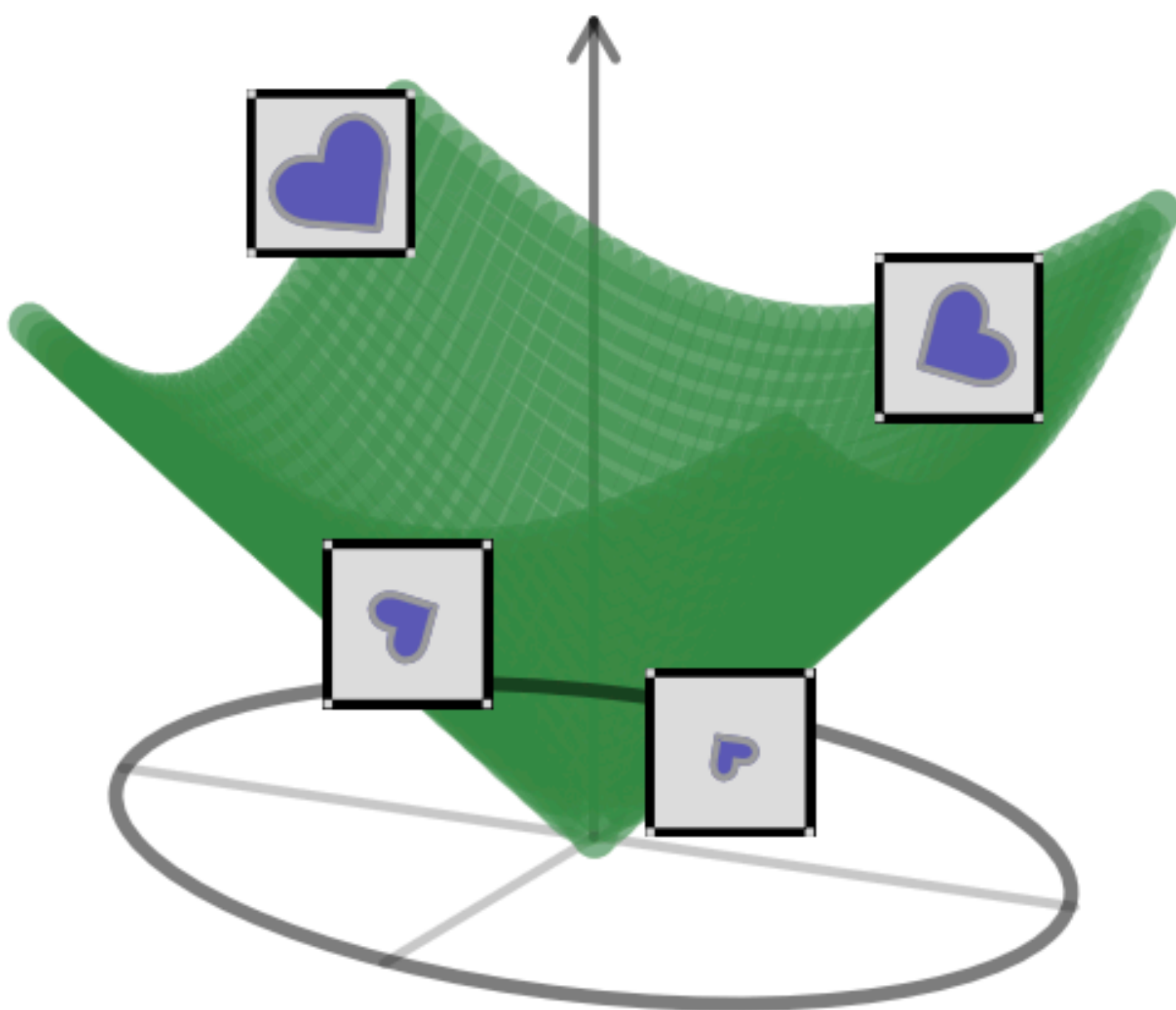


Rotation

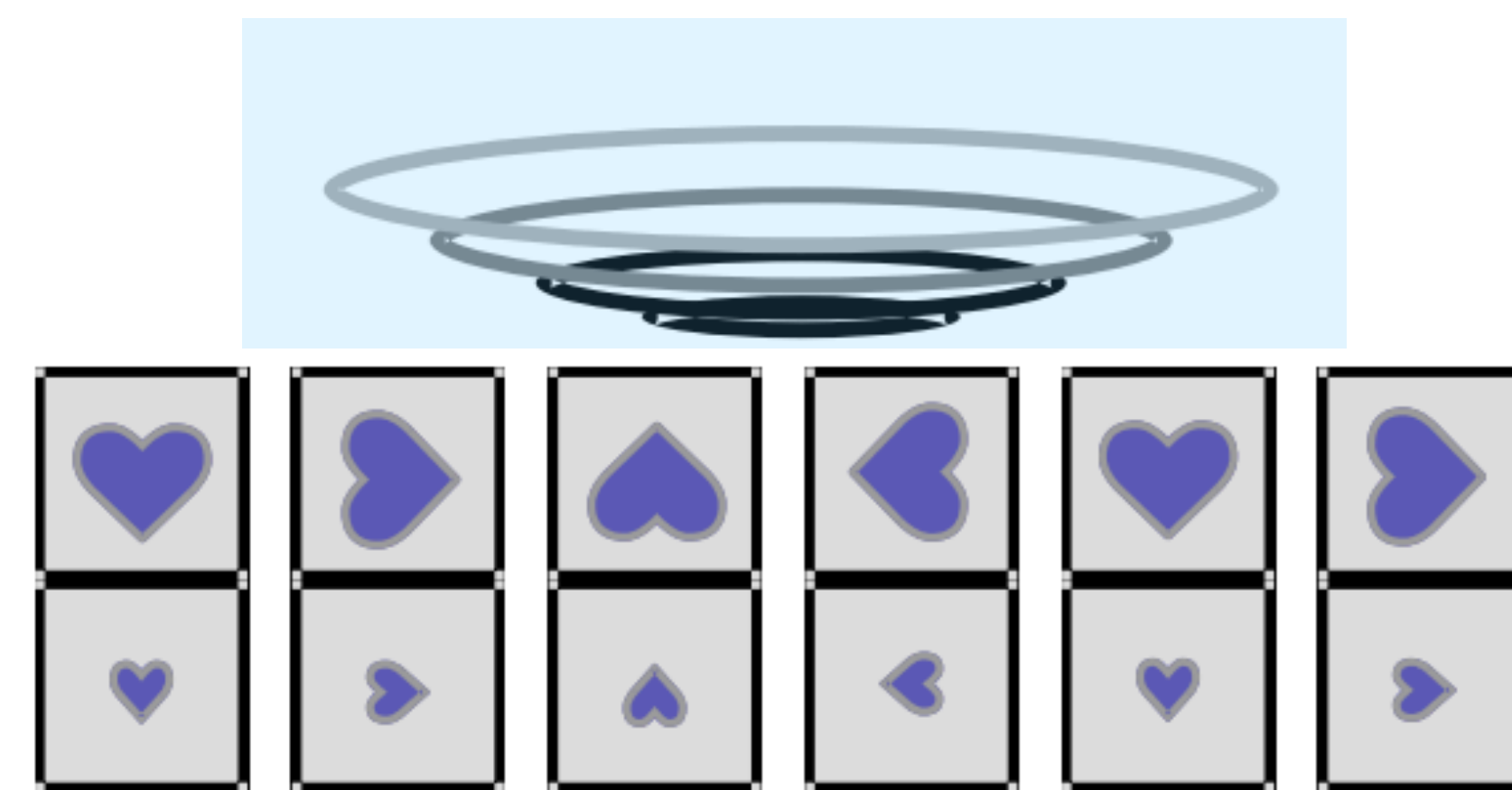






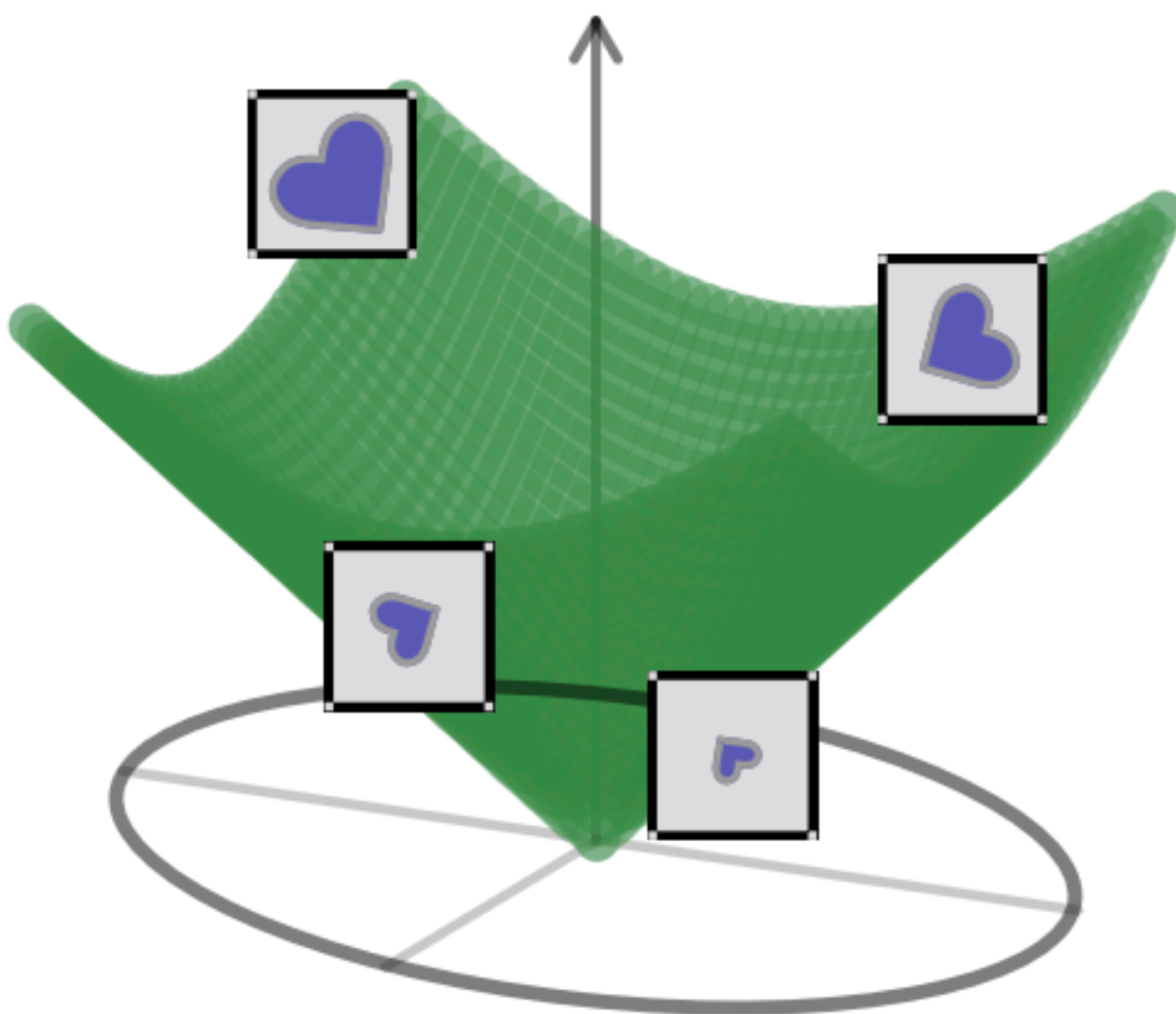


Scale

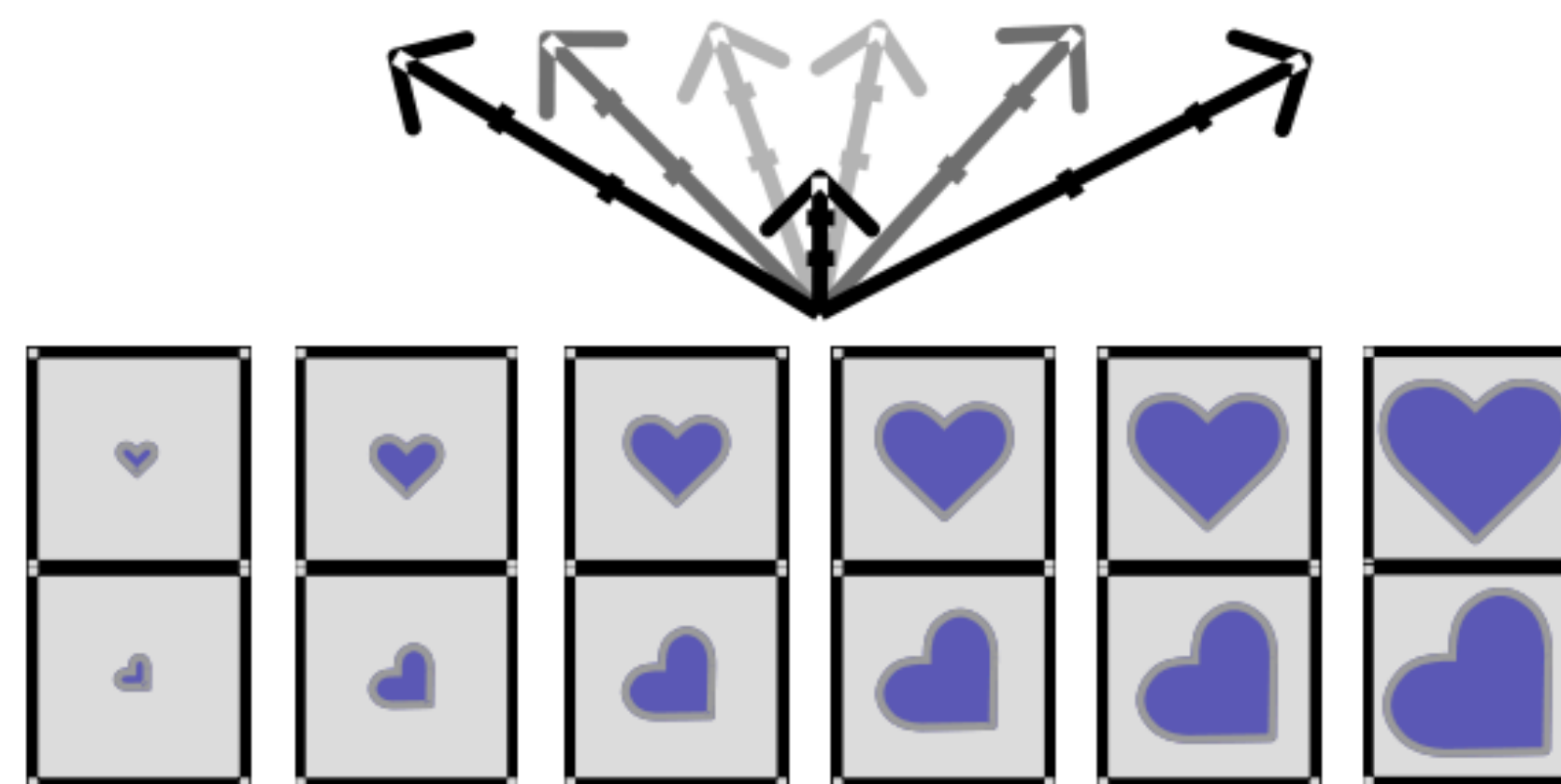


Rotation



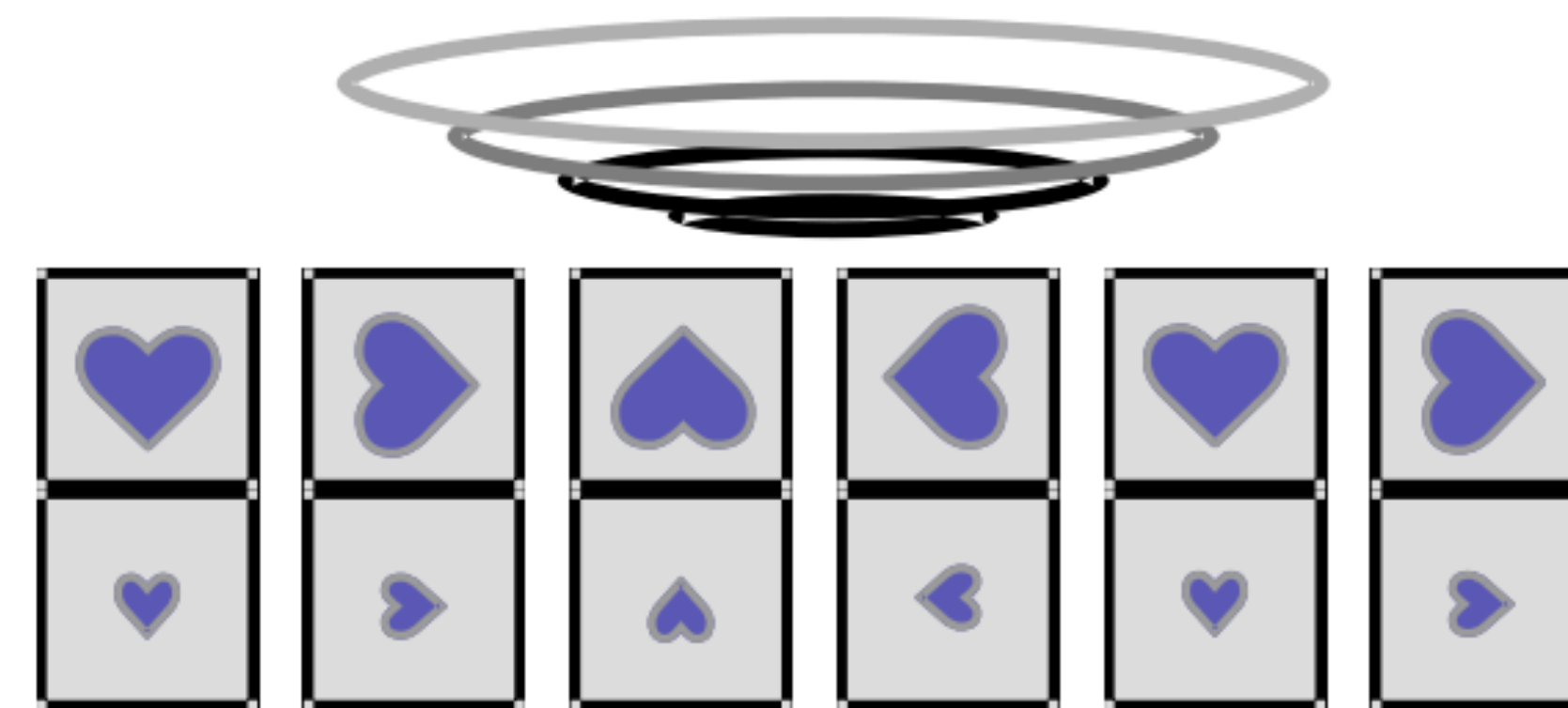


No holes



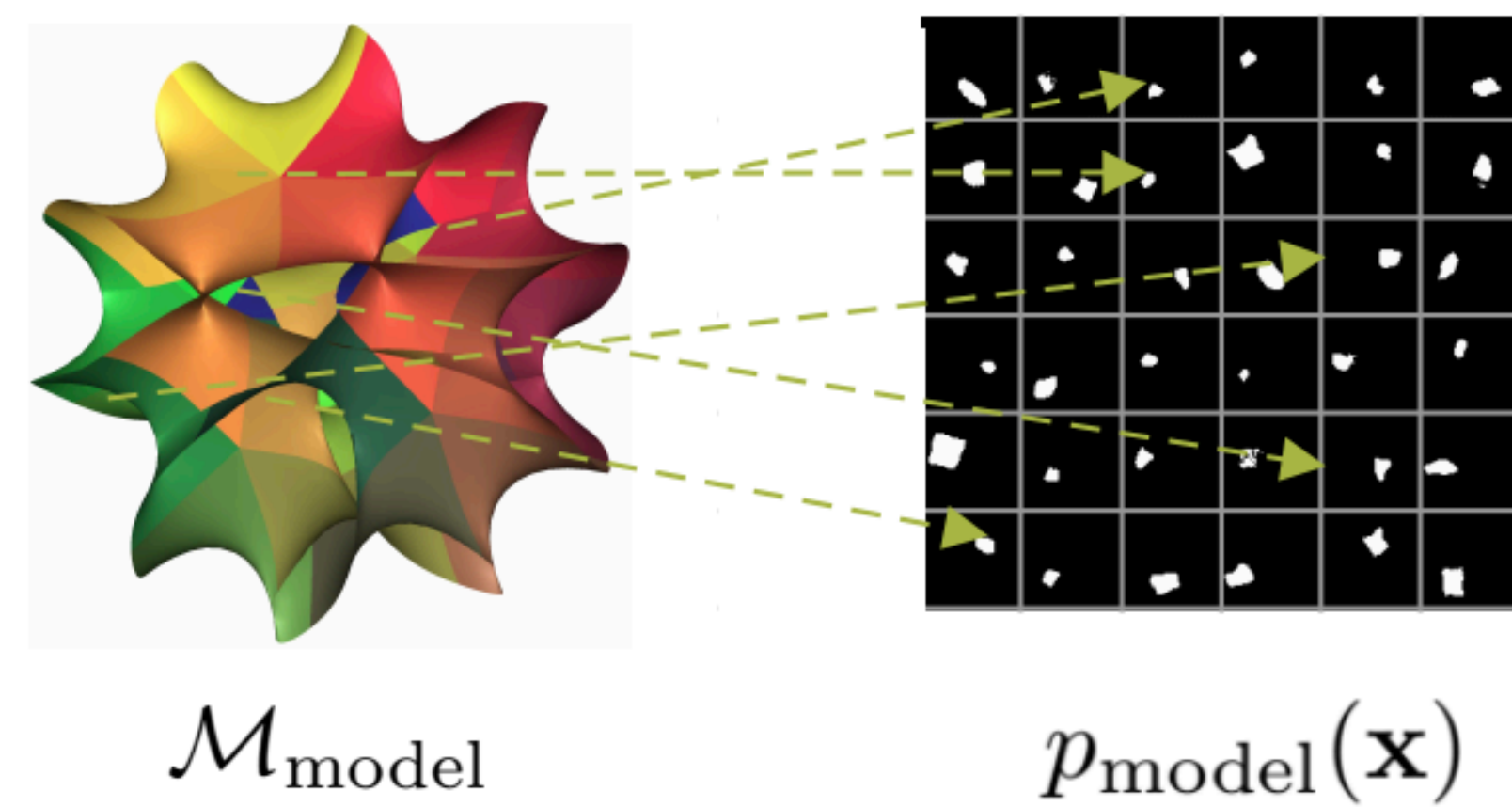
Scale

1D hole



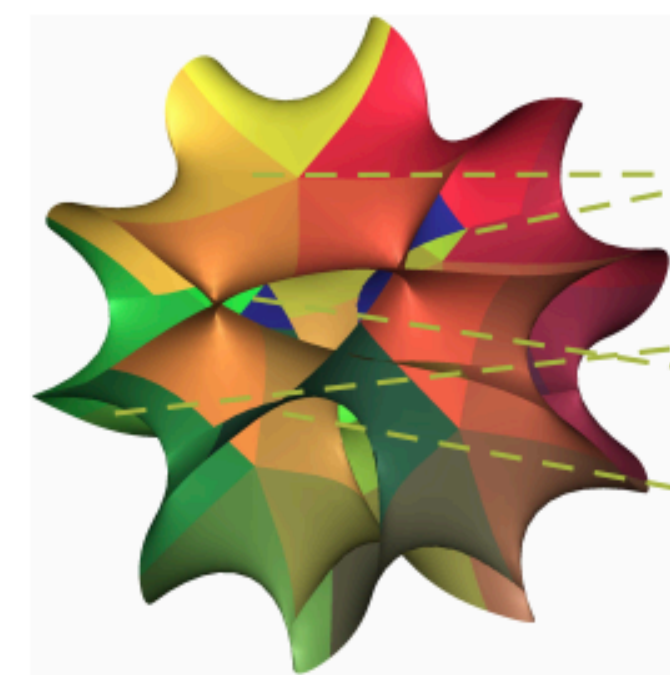
Rotation



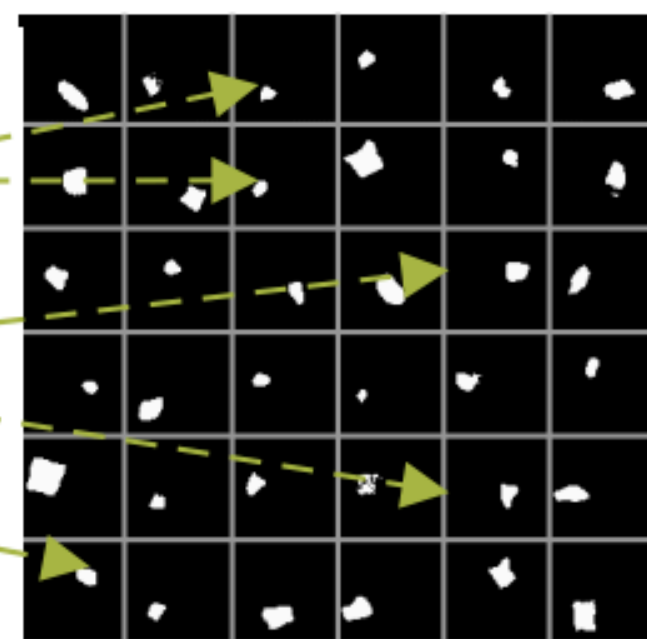


Manifold subfigure (a) is inspired by Hanson (1994)  
 Data points and corresponding simplices subfigures (b)-(c) by Khrulkov and Oseledets (2018)





$\mathcal{M}_{\text{model}}$



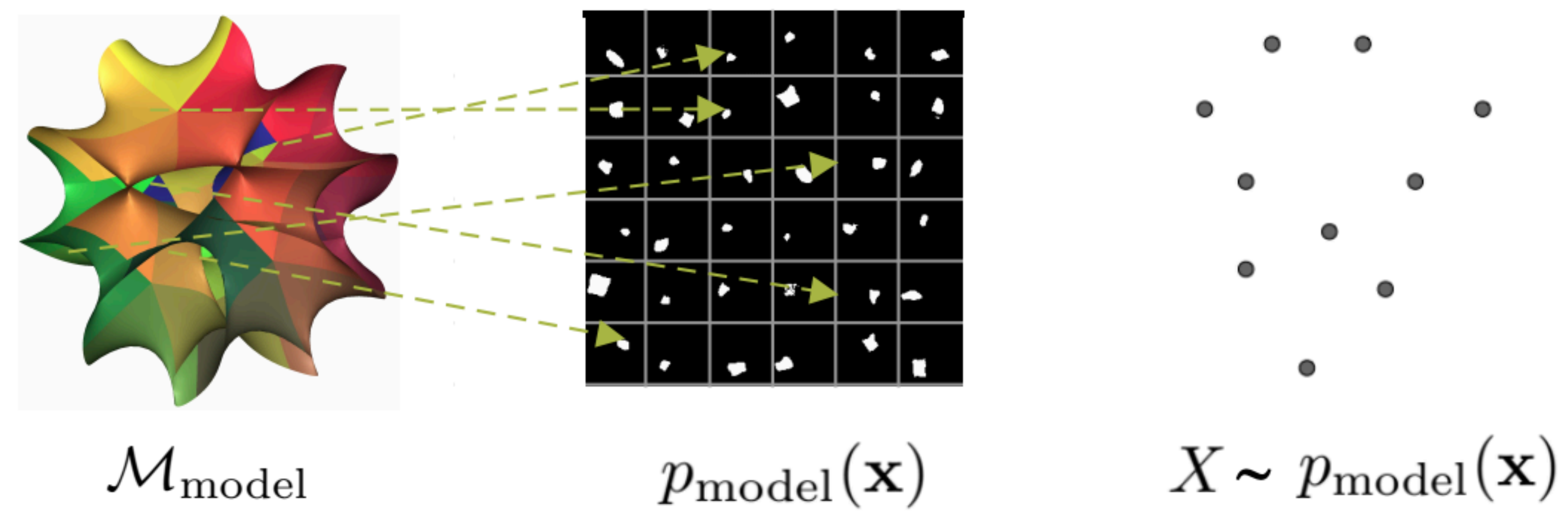
$p_{\text{model}}(\mathbf{x})$



Manifold subfigure (a) is inspired by Hanson (1994)

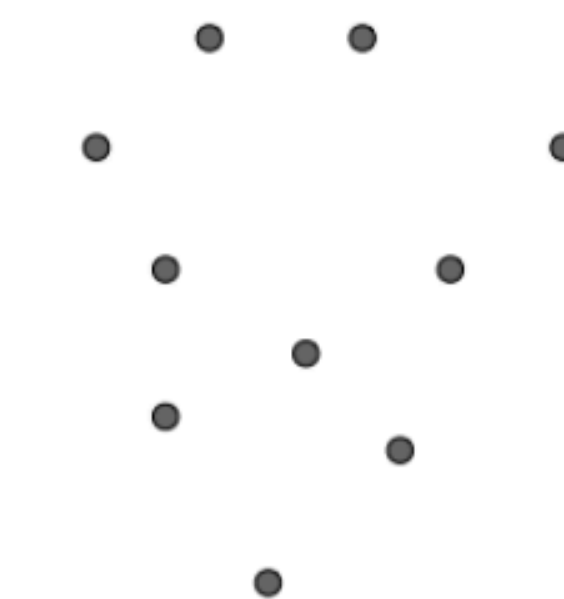
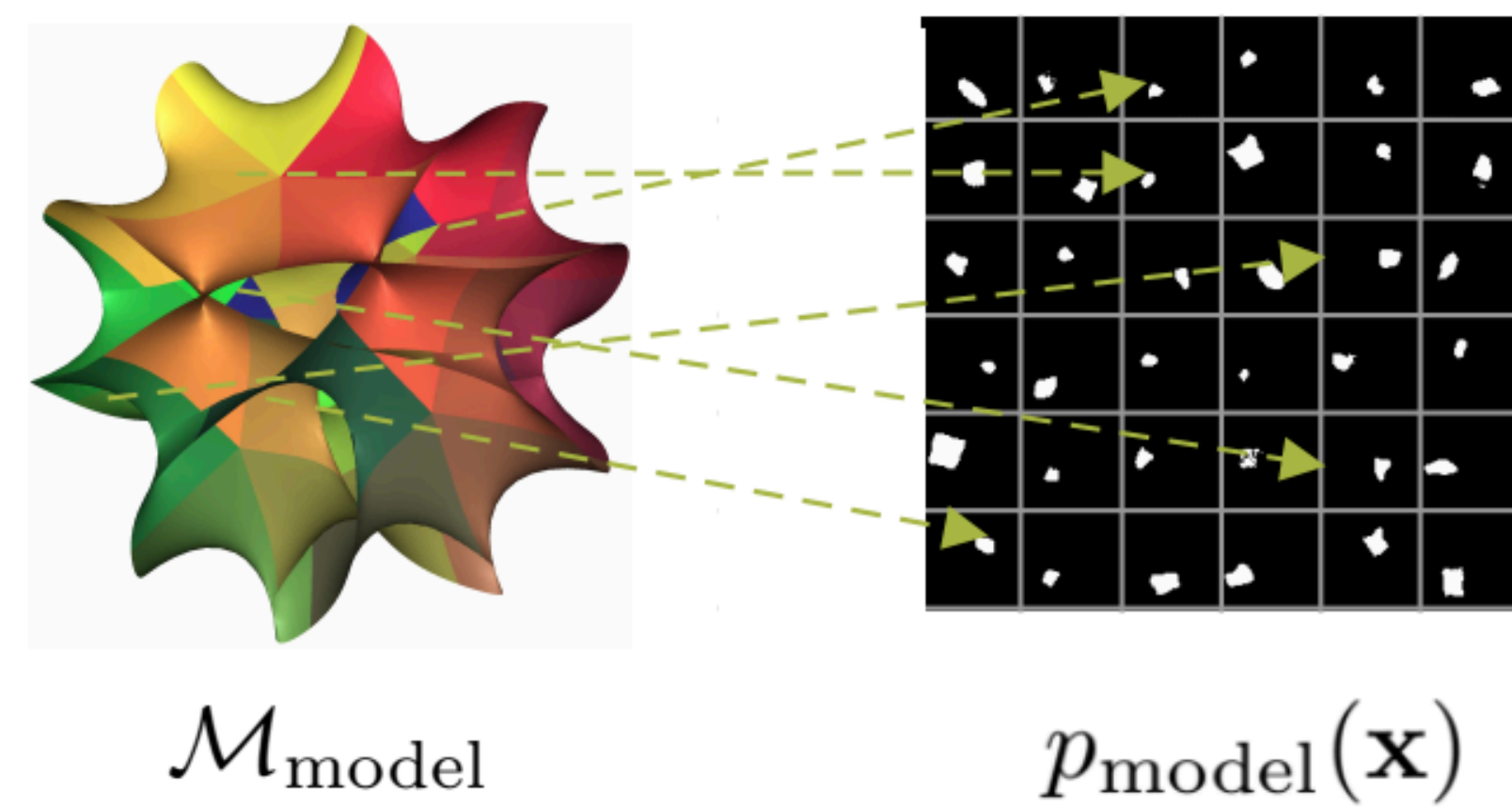
Data points and corresponding simplices subfigures (b)-(c) by Khrulkov and Oseledets (2018)



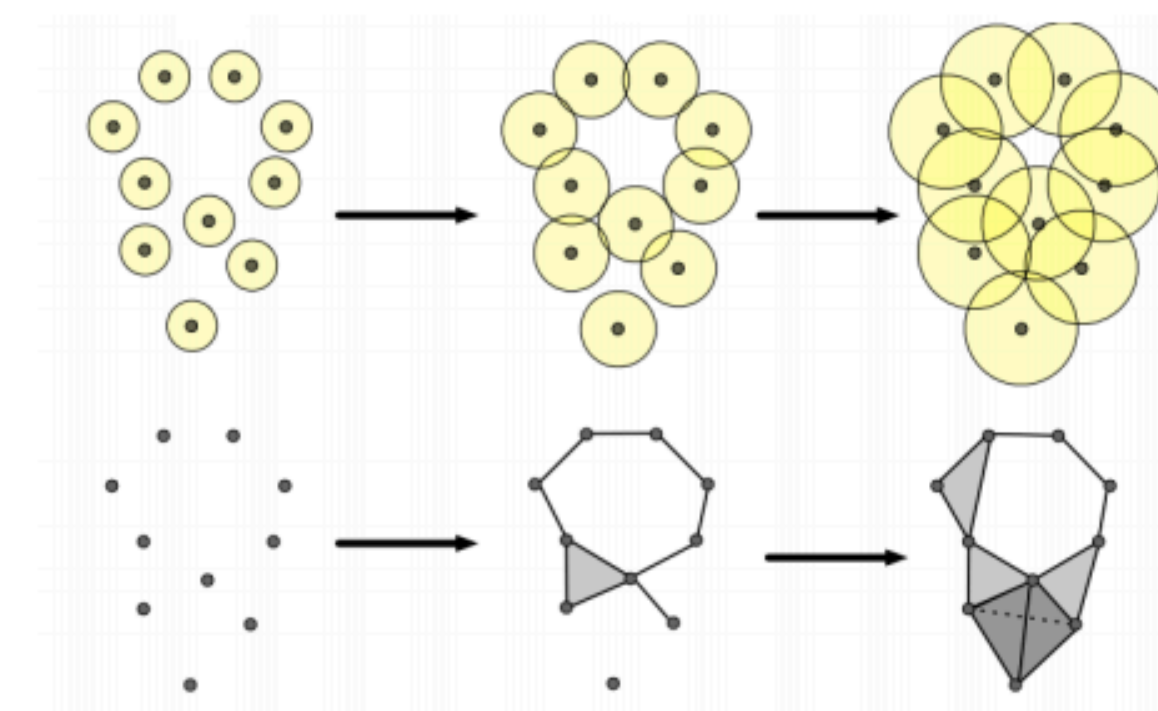


Manifold subfigure (a) is inspired by Hanson (1994)  
 Data points and corresponding simplices subfigures (b)-(c) by Khrulkov and Oseledets (2018)



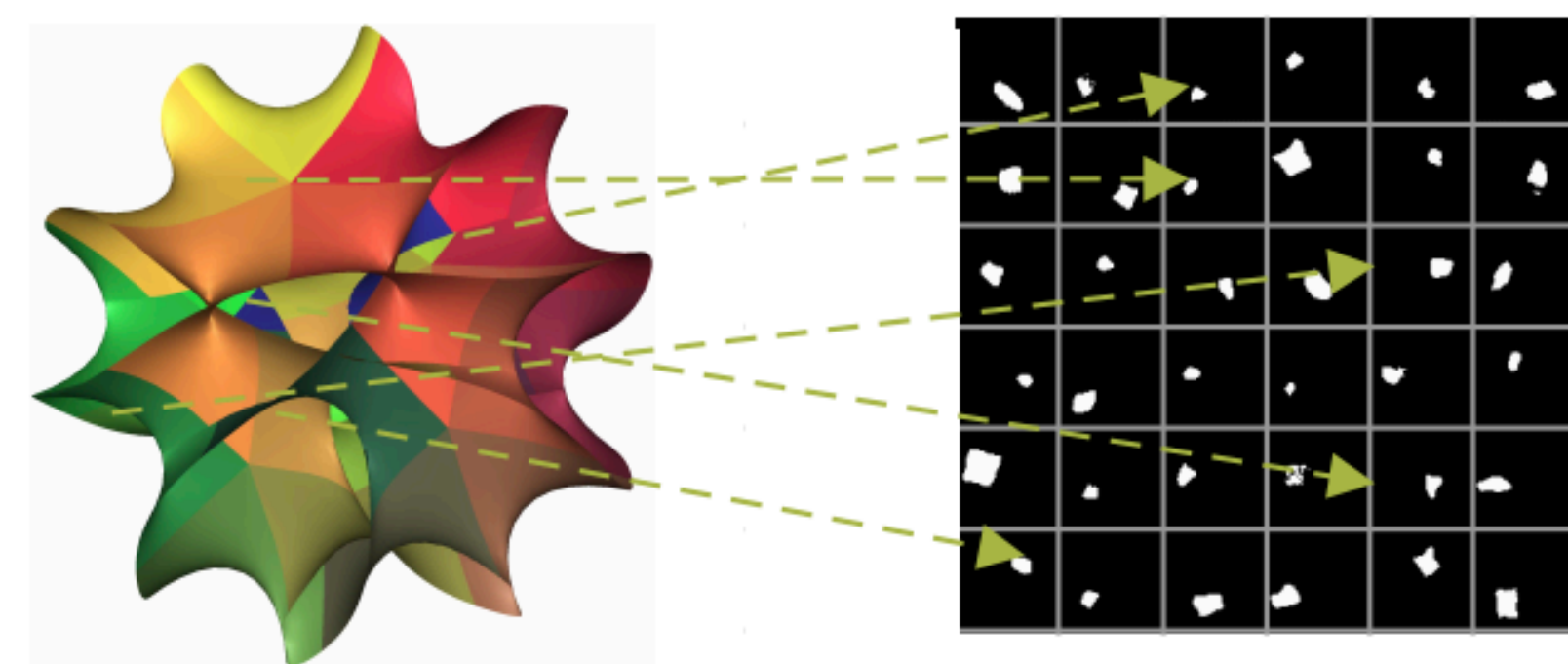


$X \sim p_{\text{model}}(\mathbf{x})$

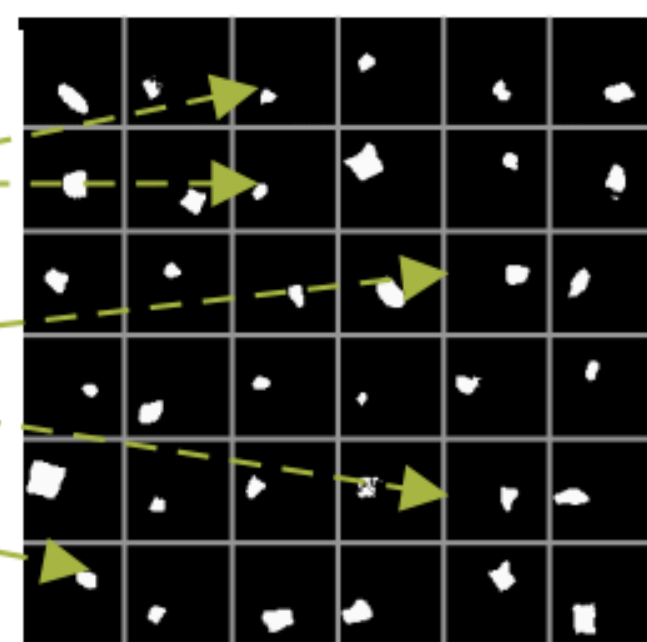


Simplices, increasing proximity

Manifold subfigure (a) is inspired by Hanson (1994)  
 Data points and corresponding simplices subfigures (b)-(c) by Khrulkov and Oseledets (2018)



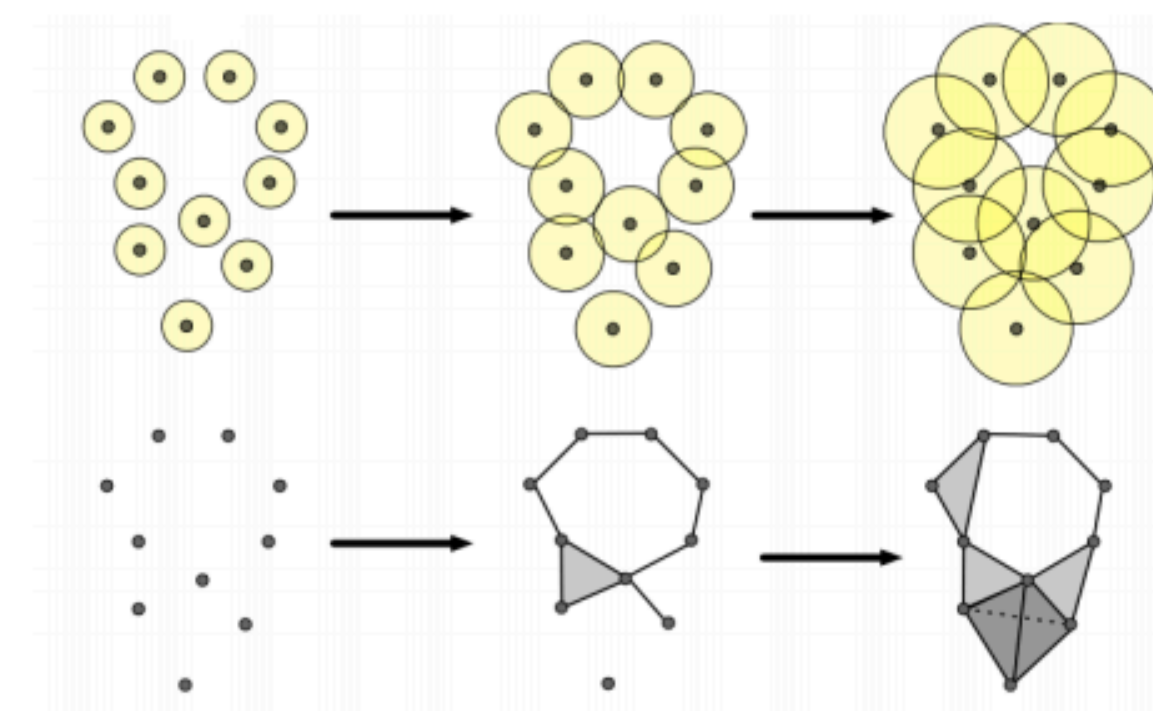
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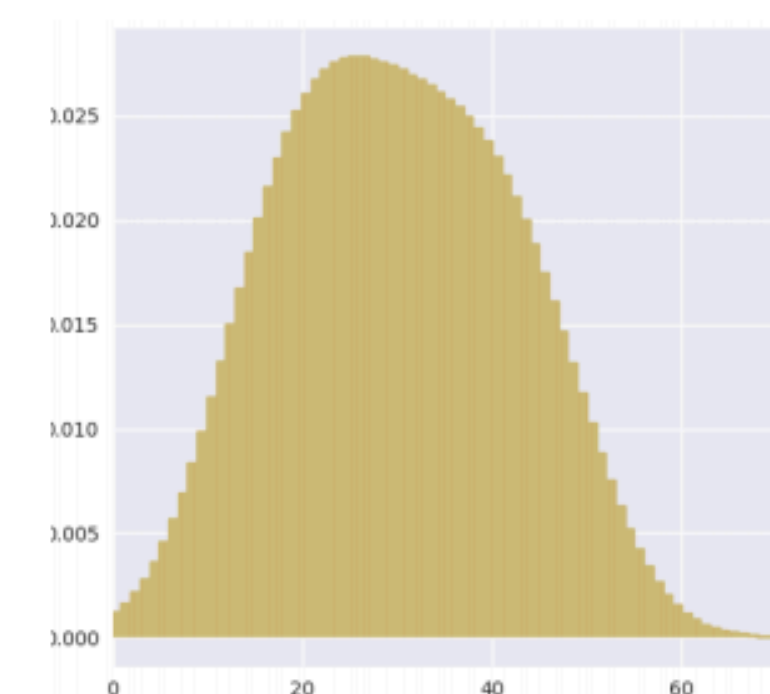
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$X \sim p_{\text{model}}(\mathbf{x})$



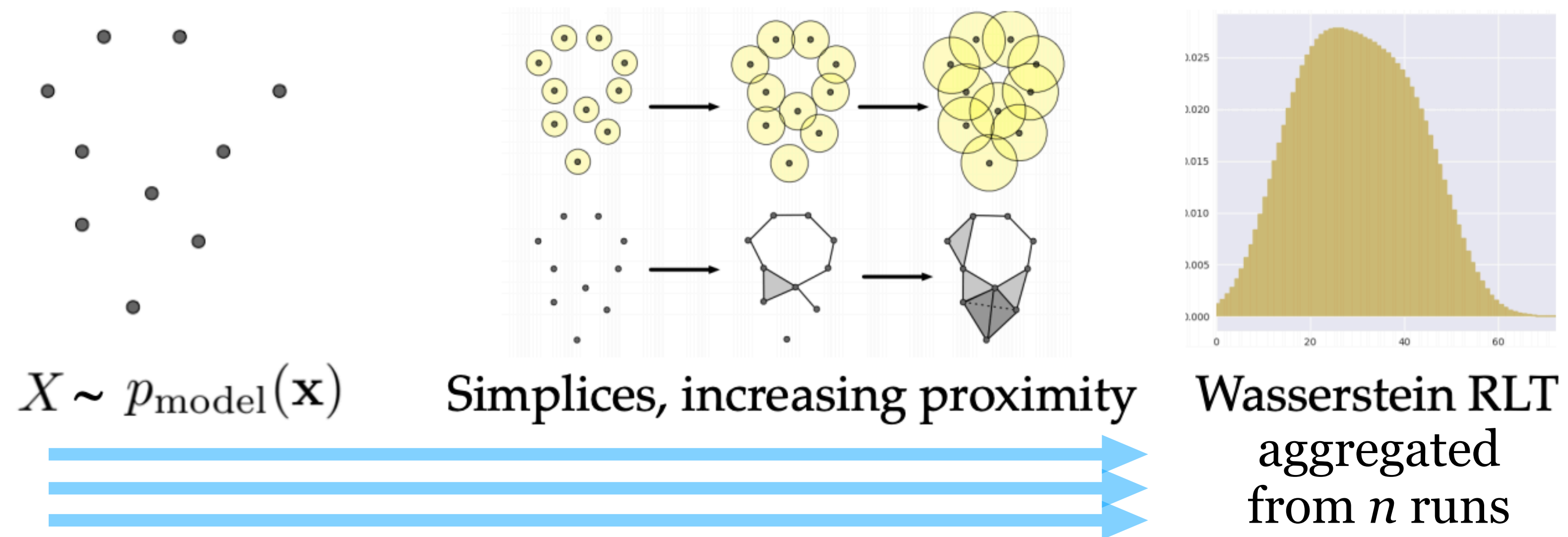
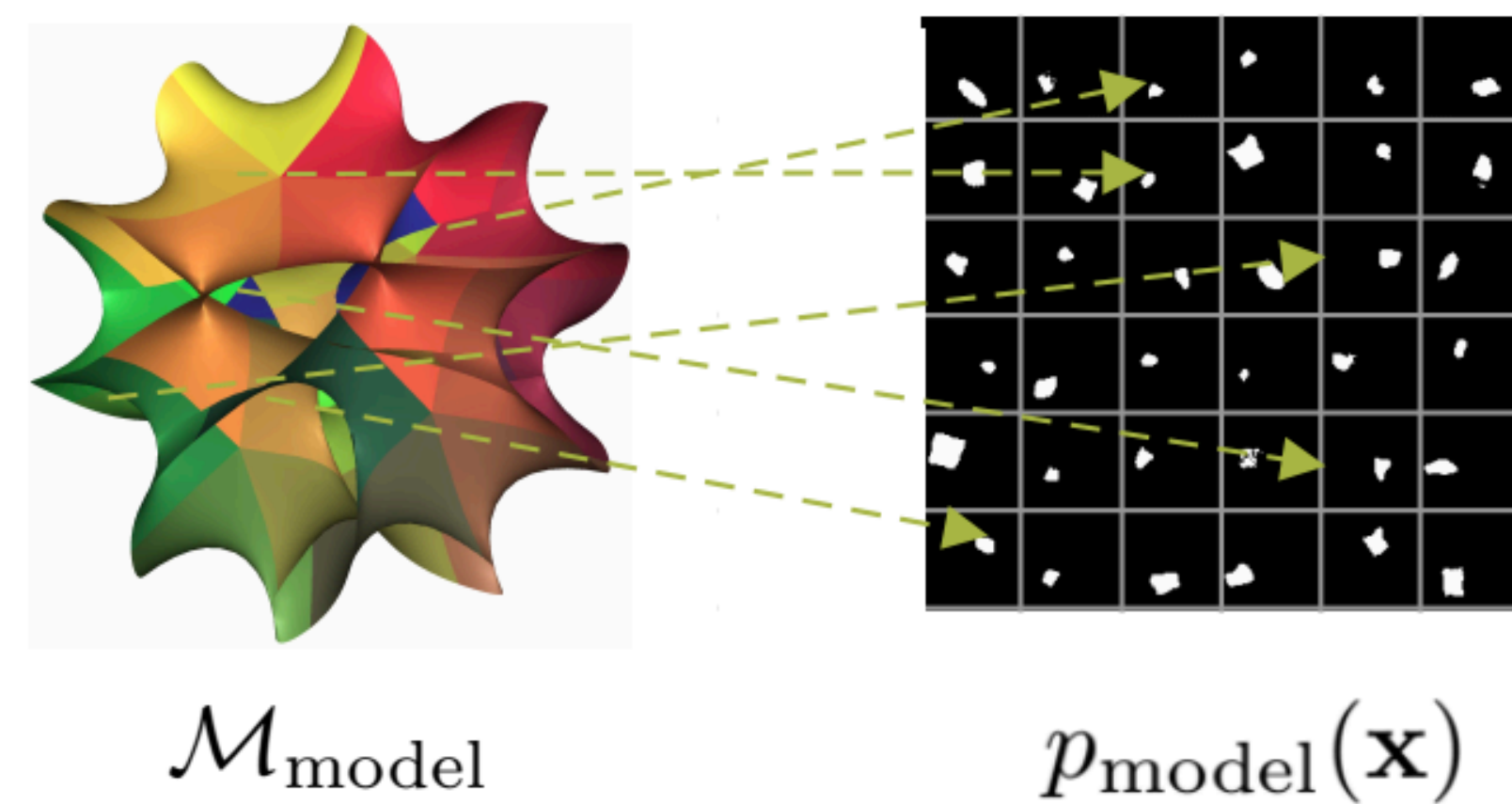
Simplices, increasing proximity



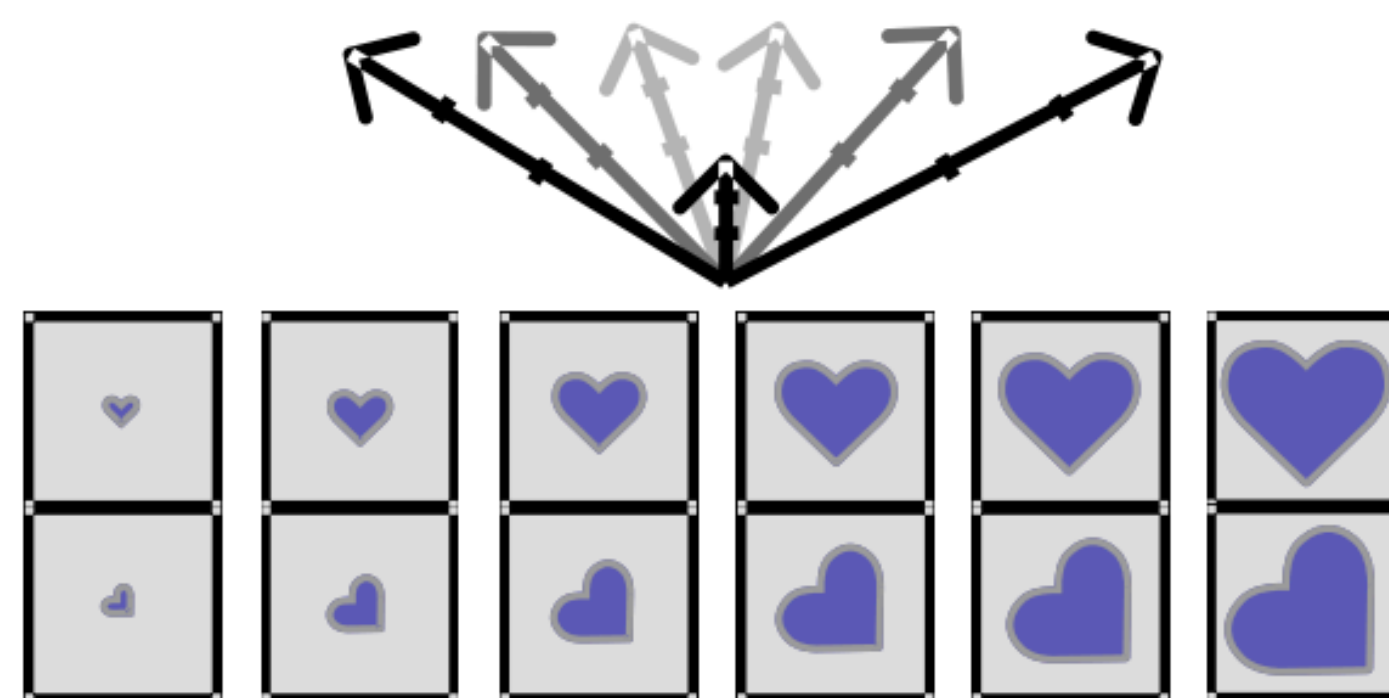
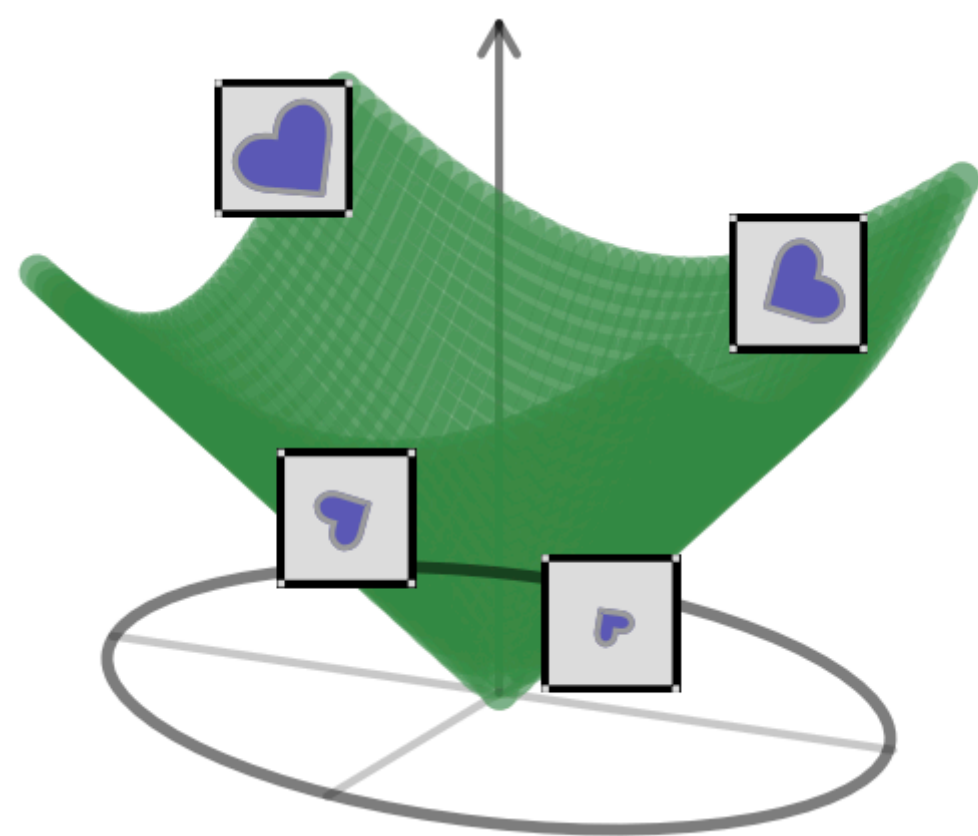
Wasserstein RLT

Manifold subfigure (a) is inspired by Hanson (1994)  
 Data points and corresponding simplices subfigures (b)-(c) by Khrulkov and Oseledets (2018)

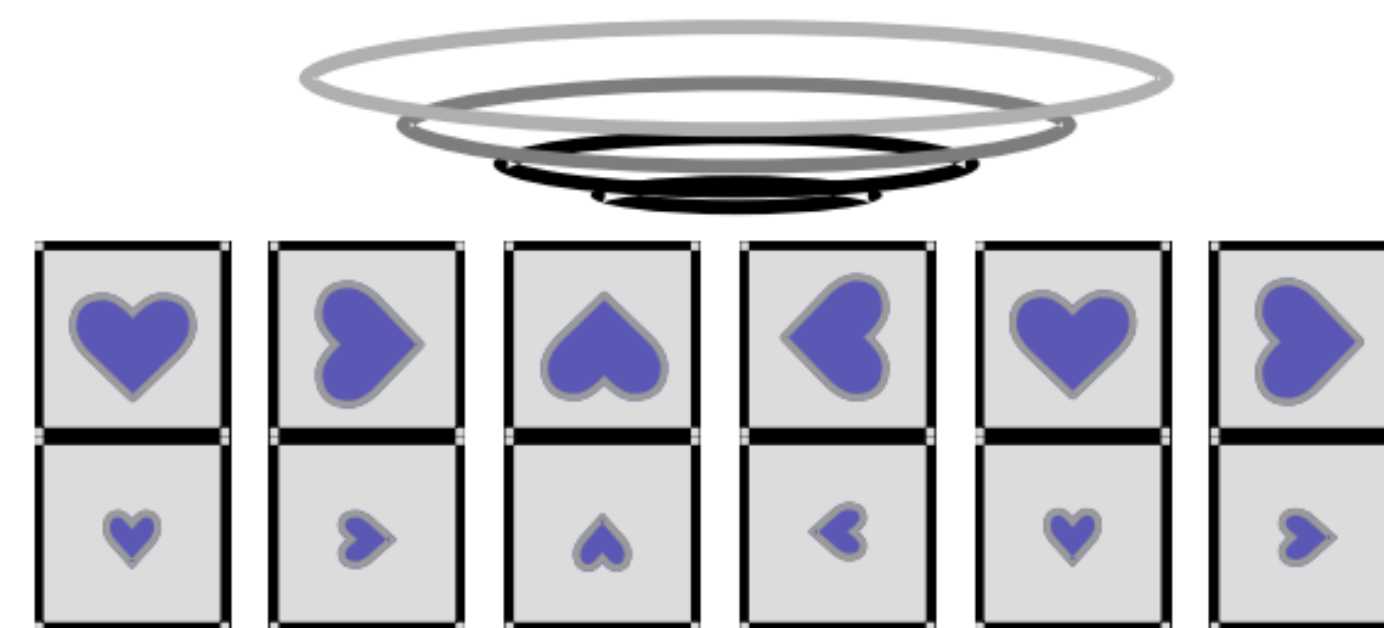




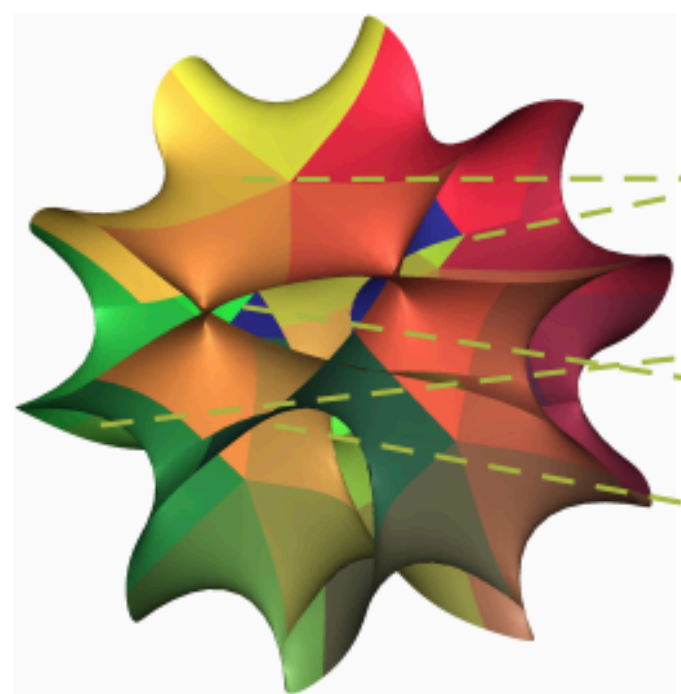
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Scale

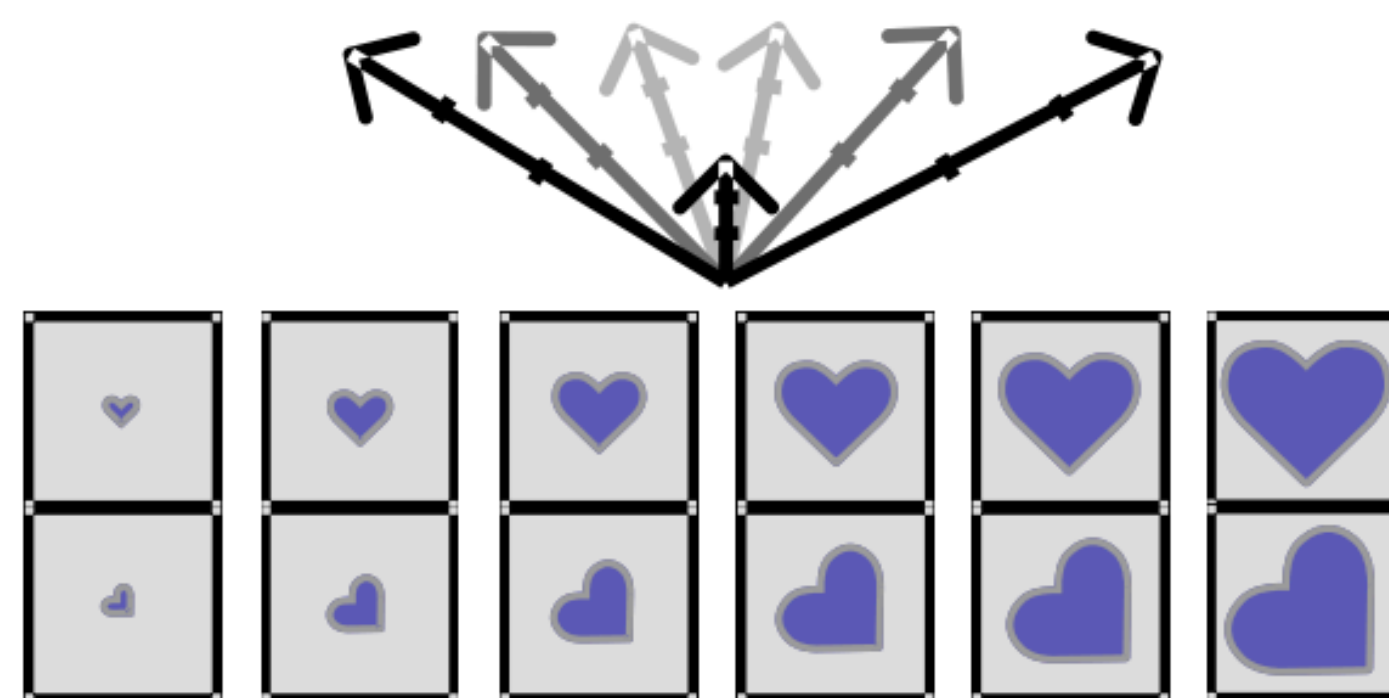
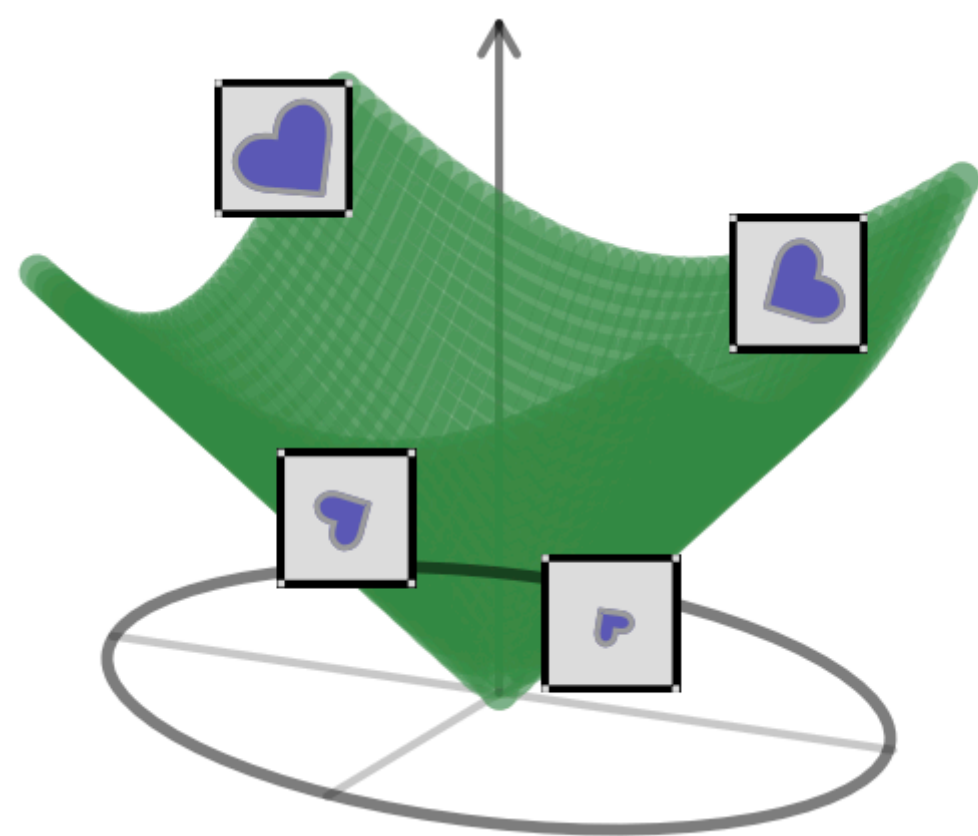


Rotation

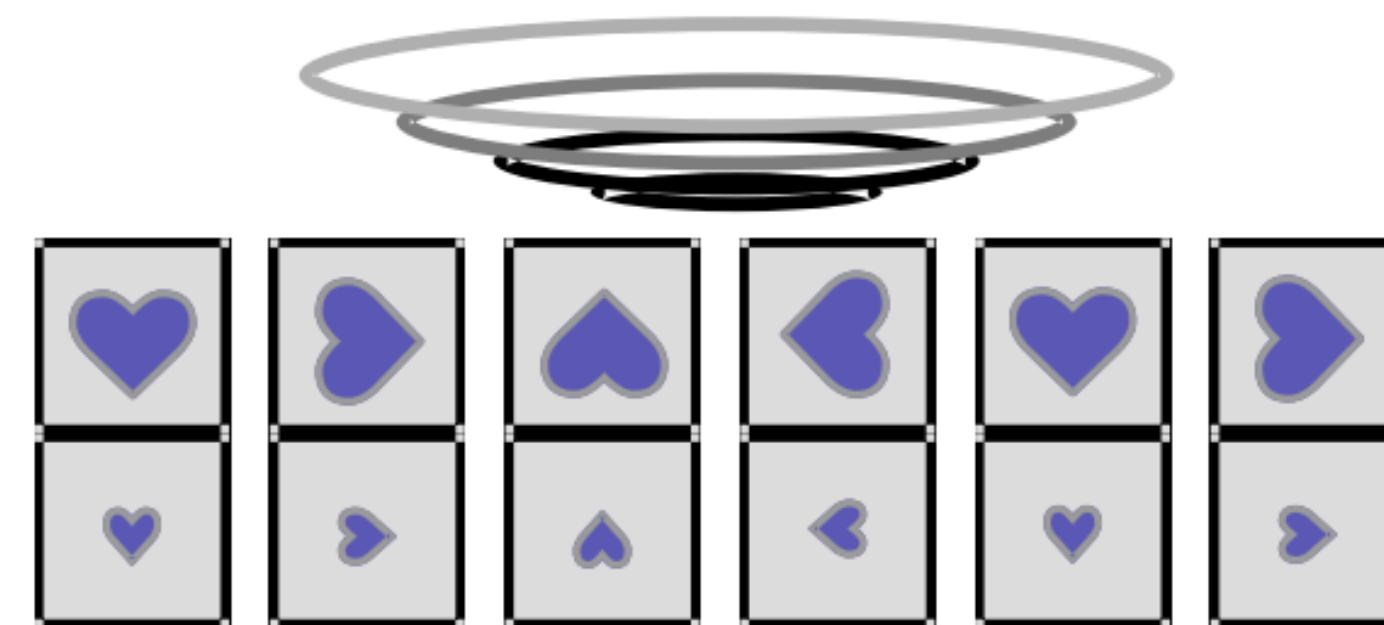


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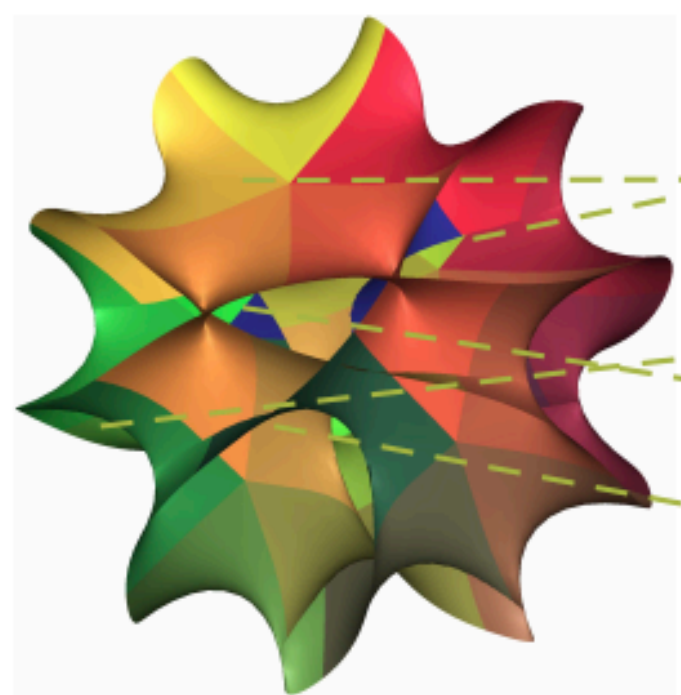




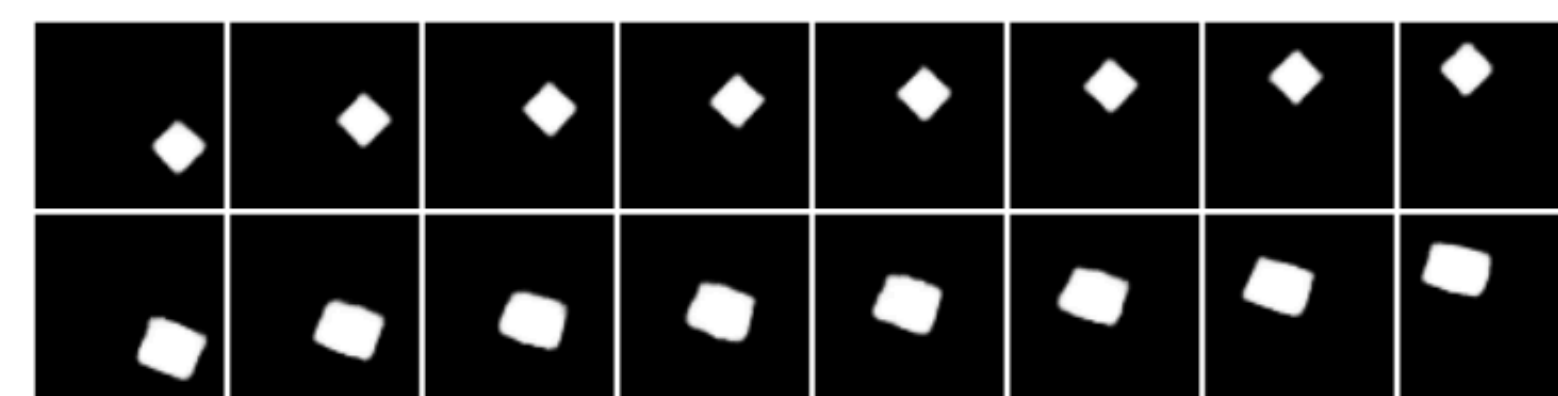
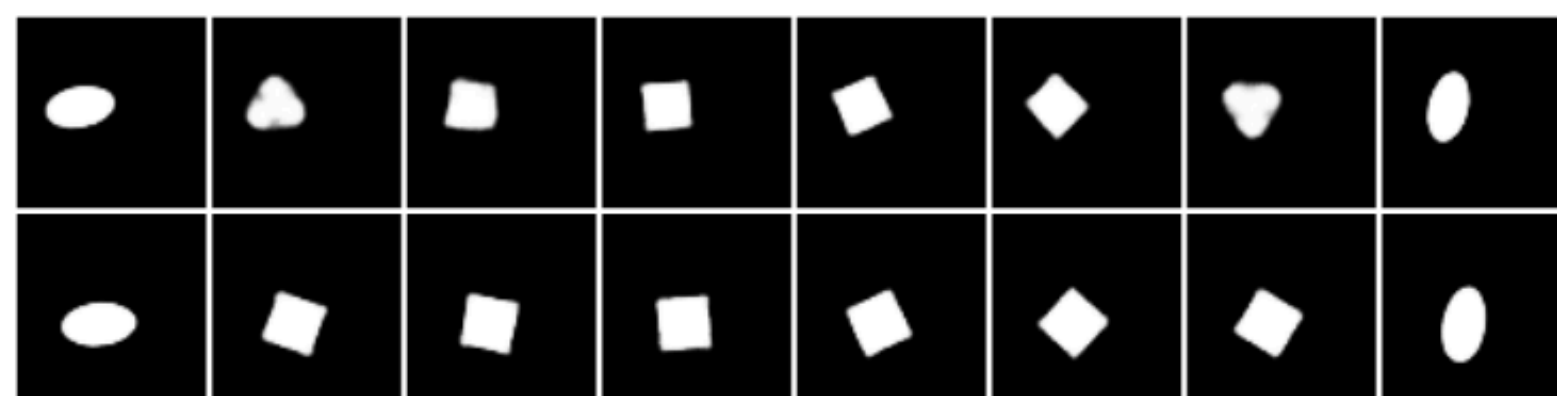
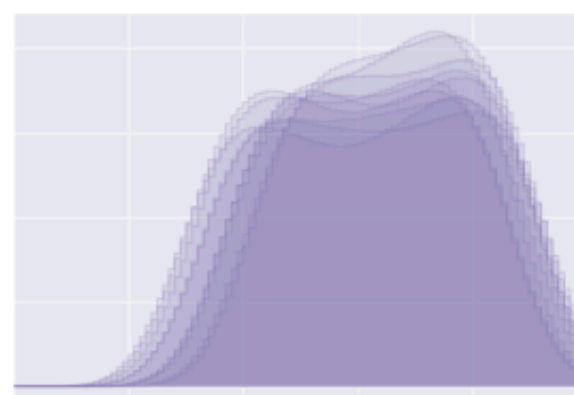
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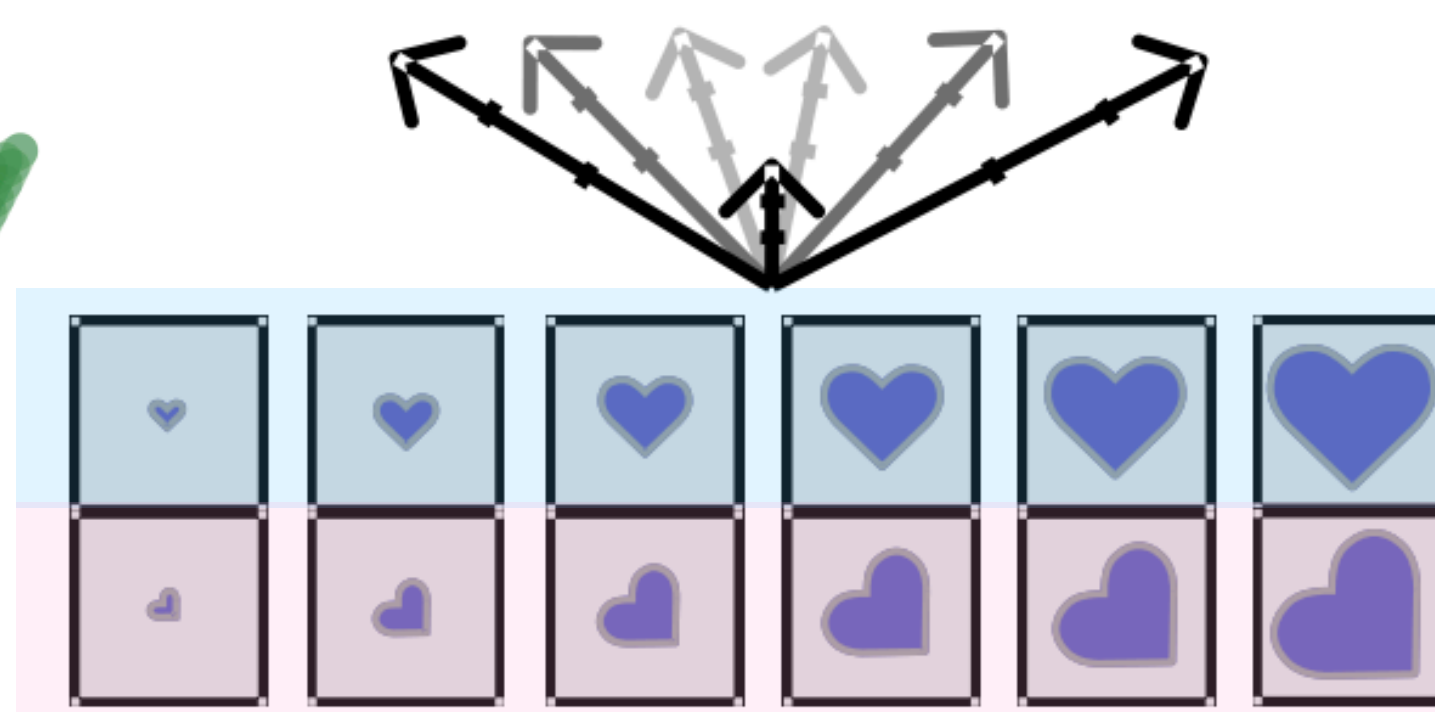
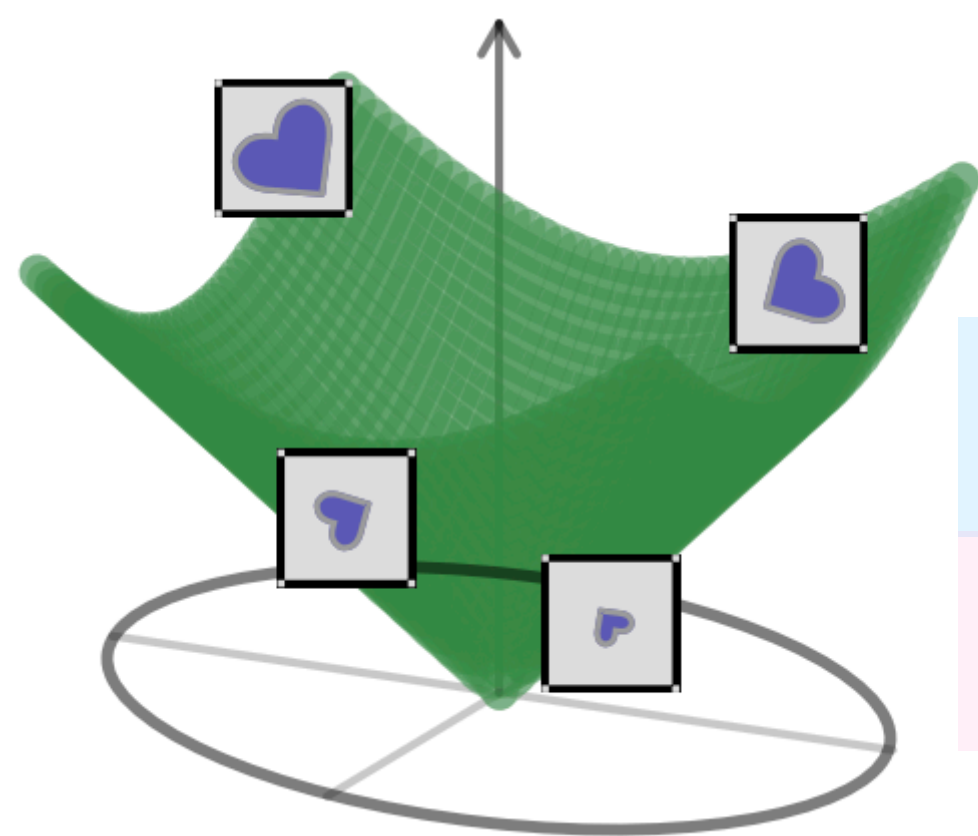


Rotation

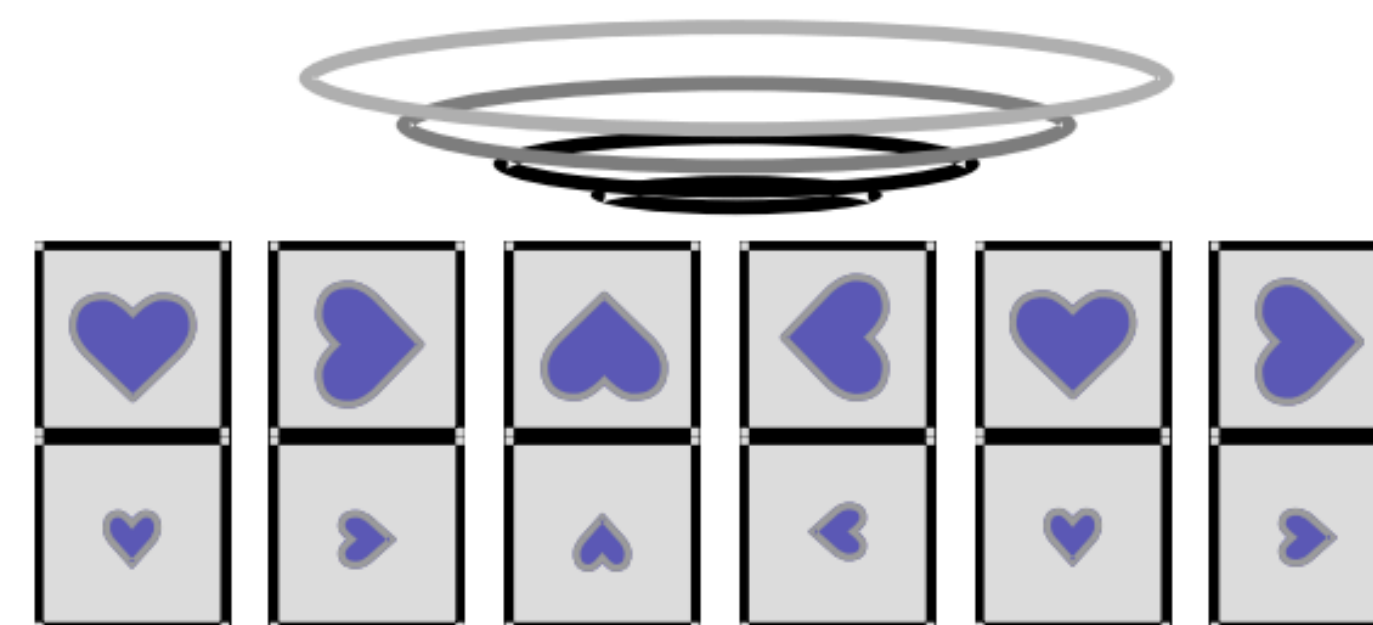


$\mathcal{M}_{\text{model}}$

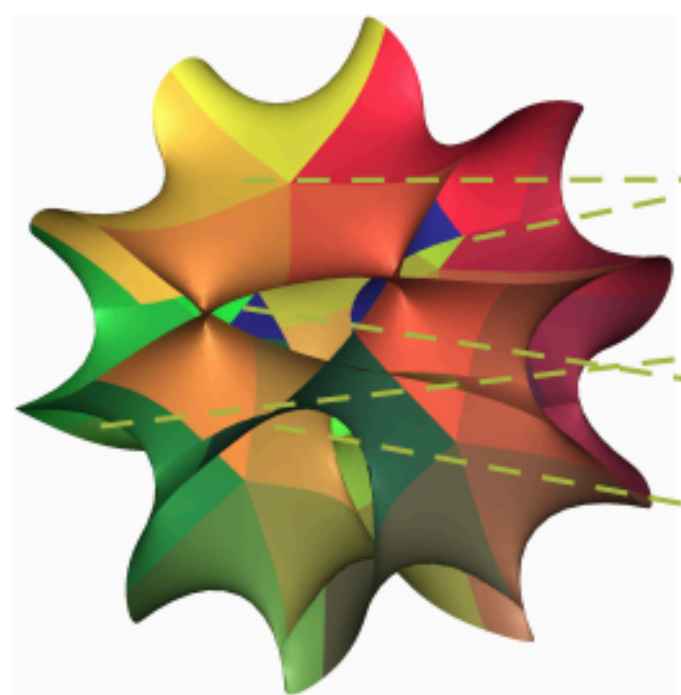




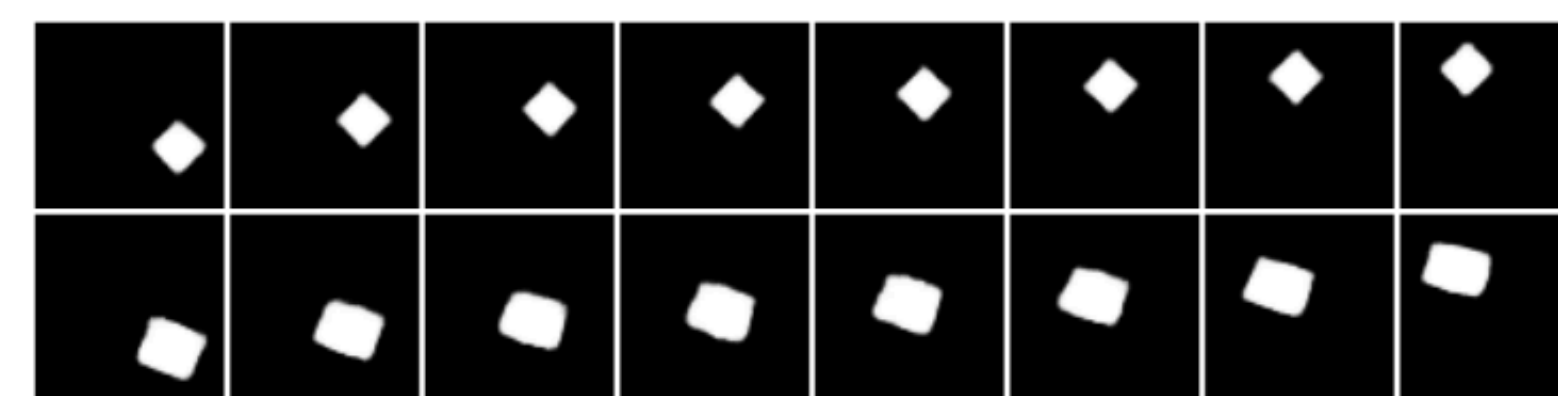
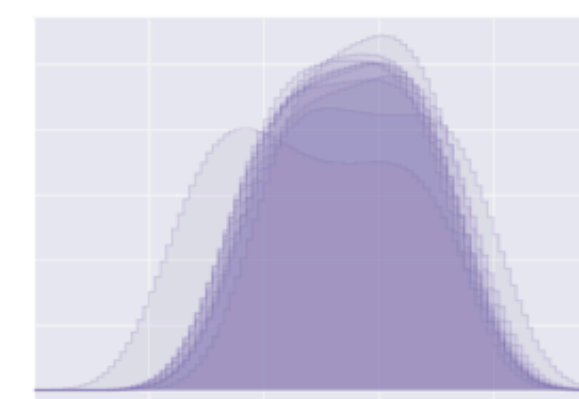
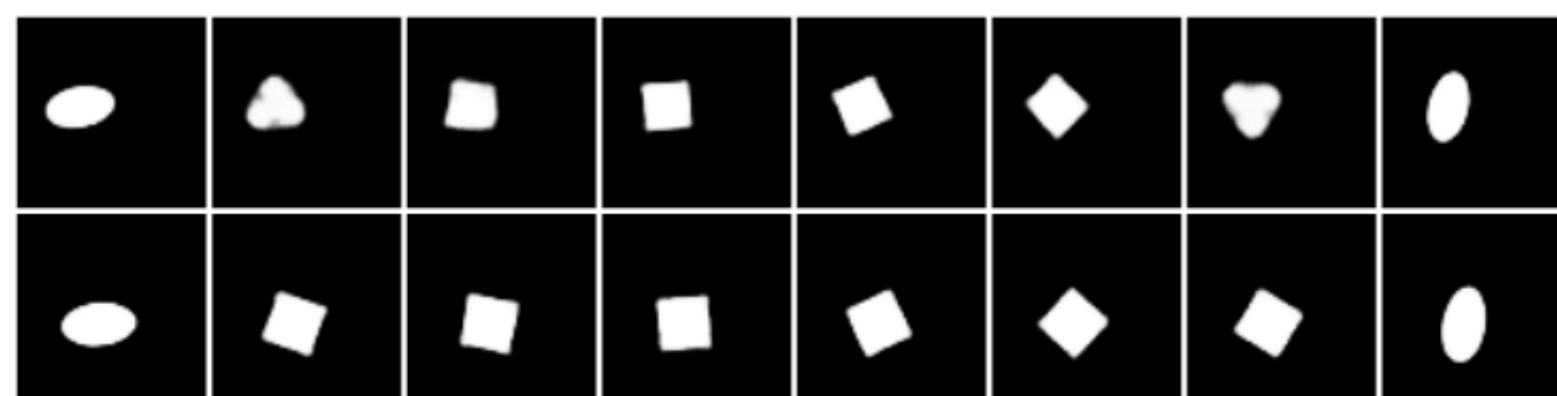
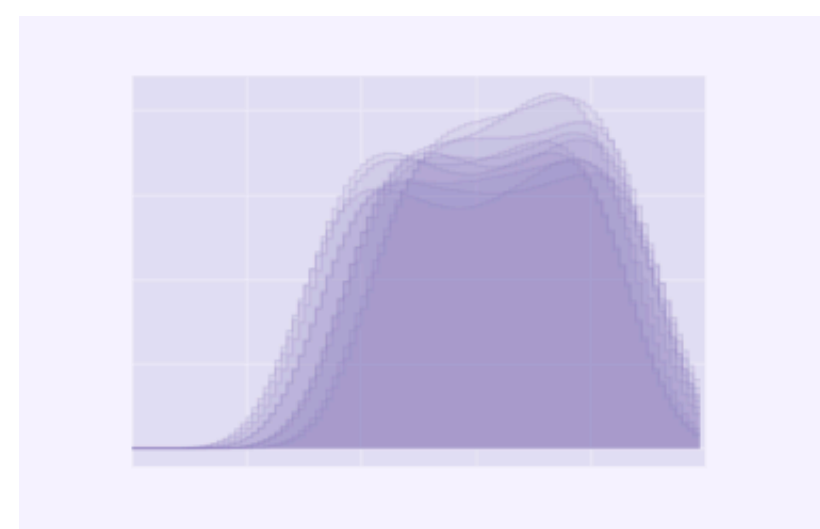
Scale



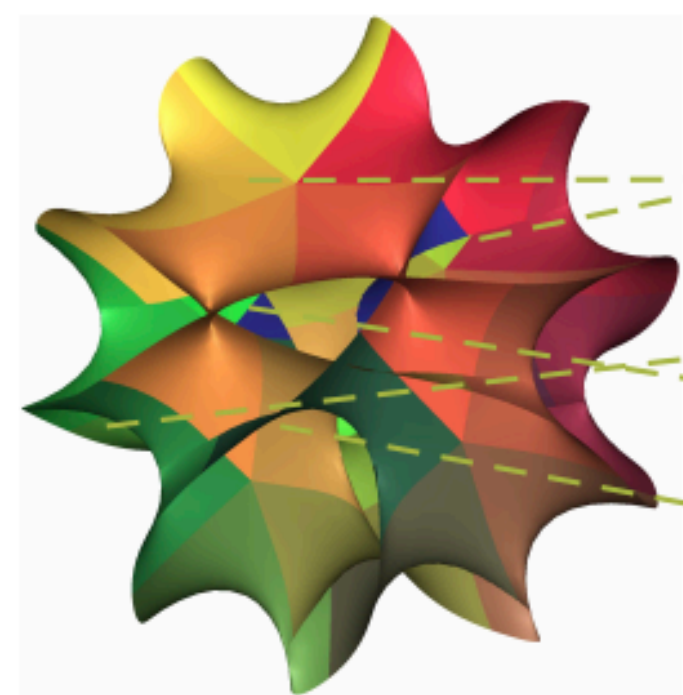
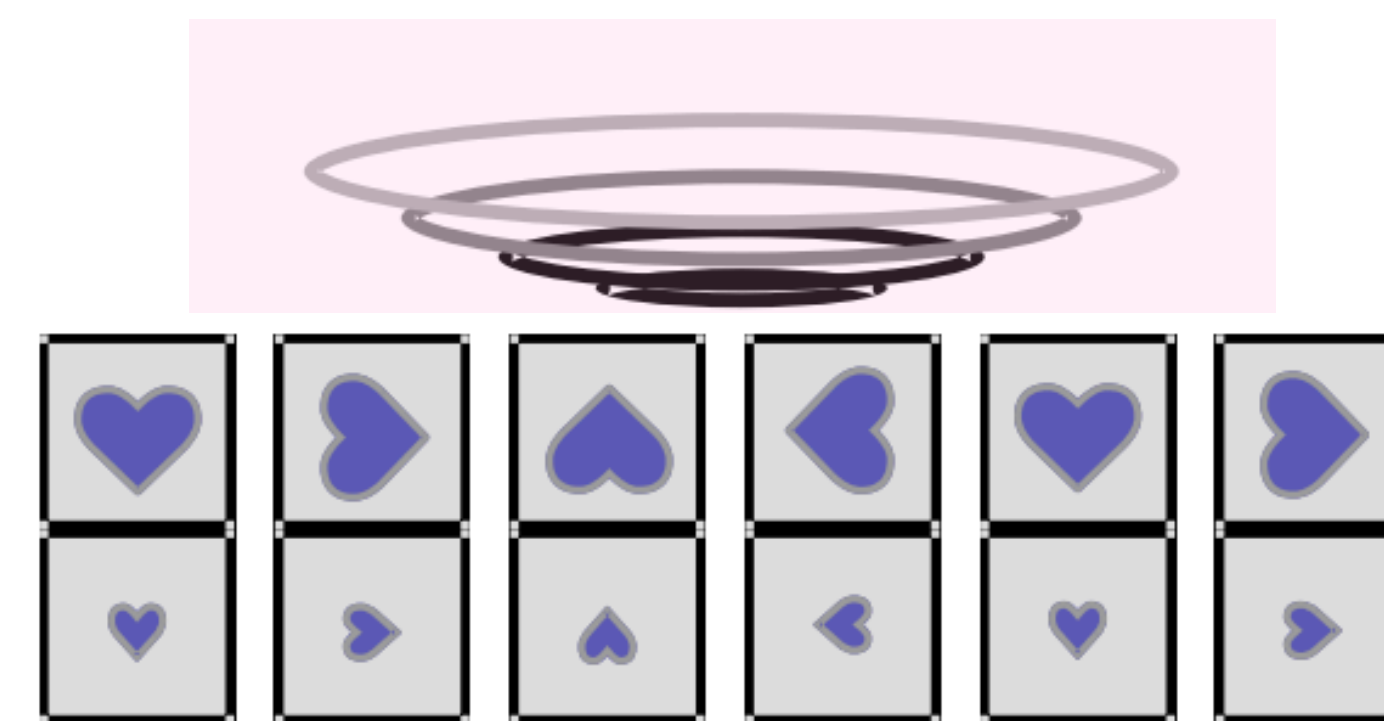
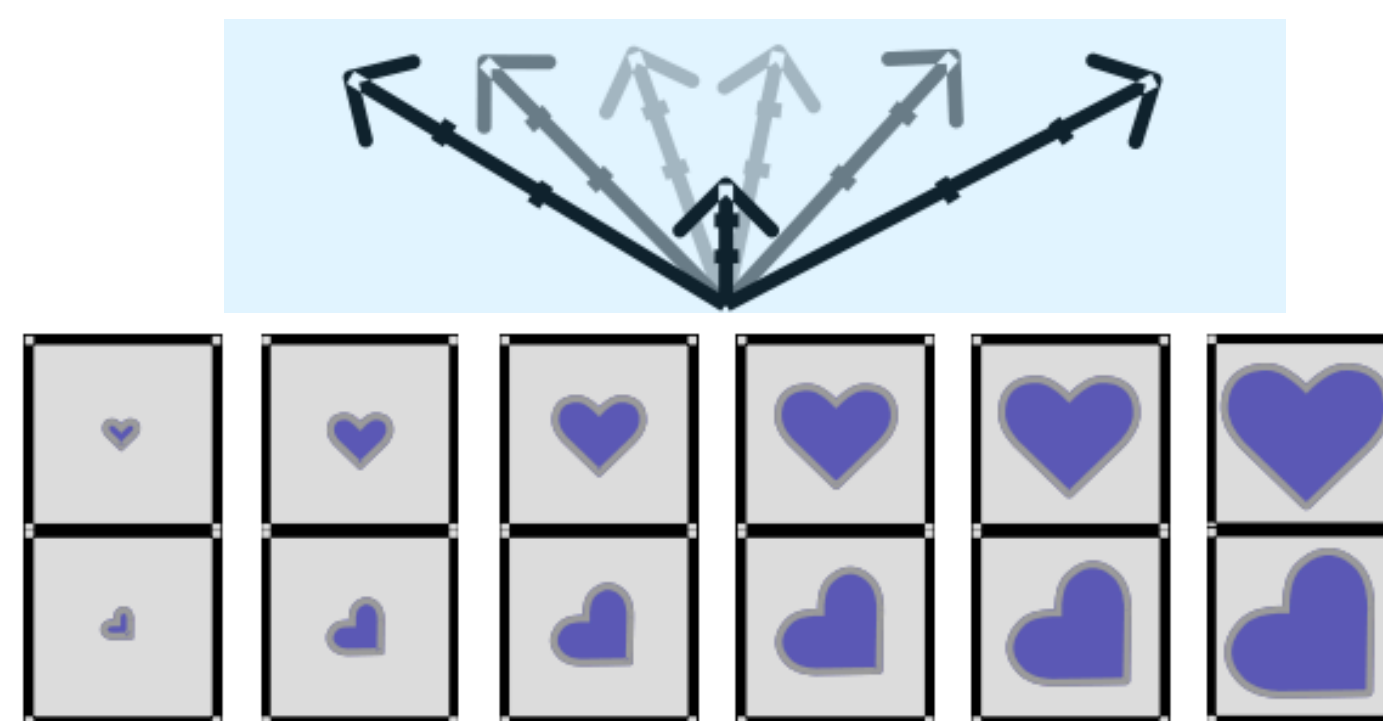
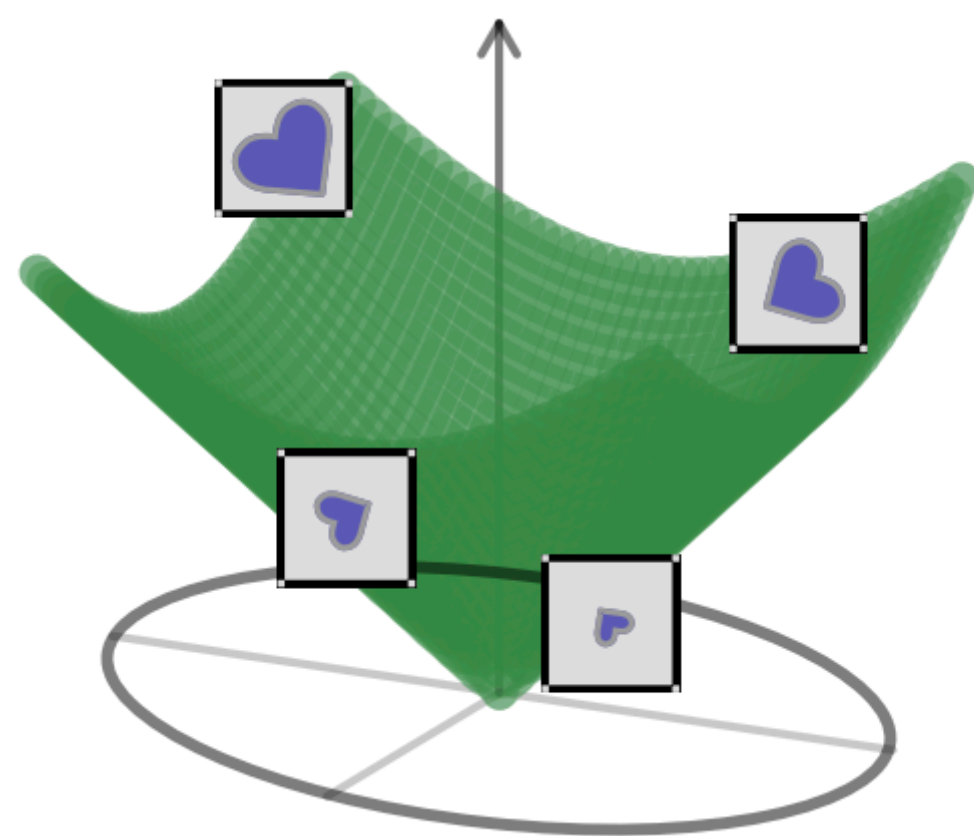
Rotation



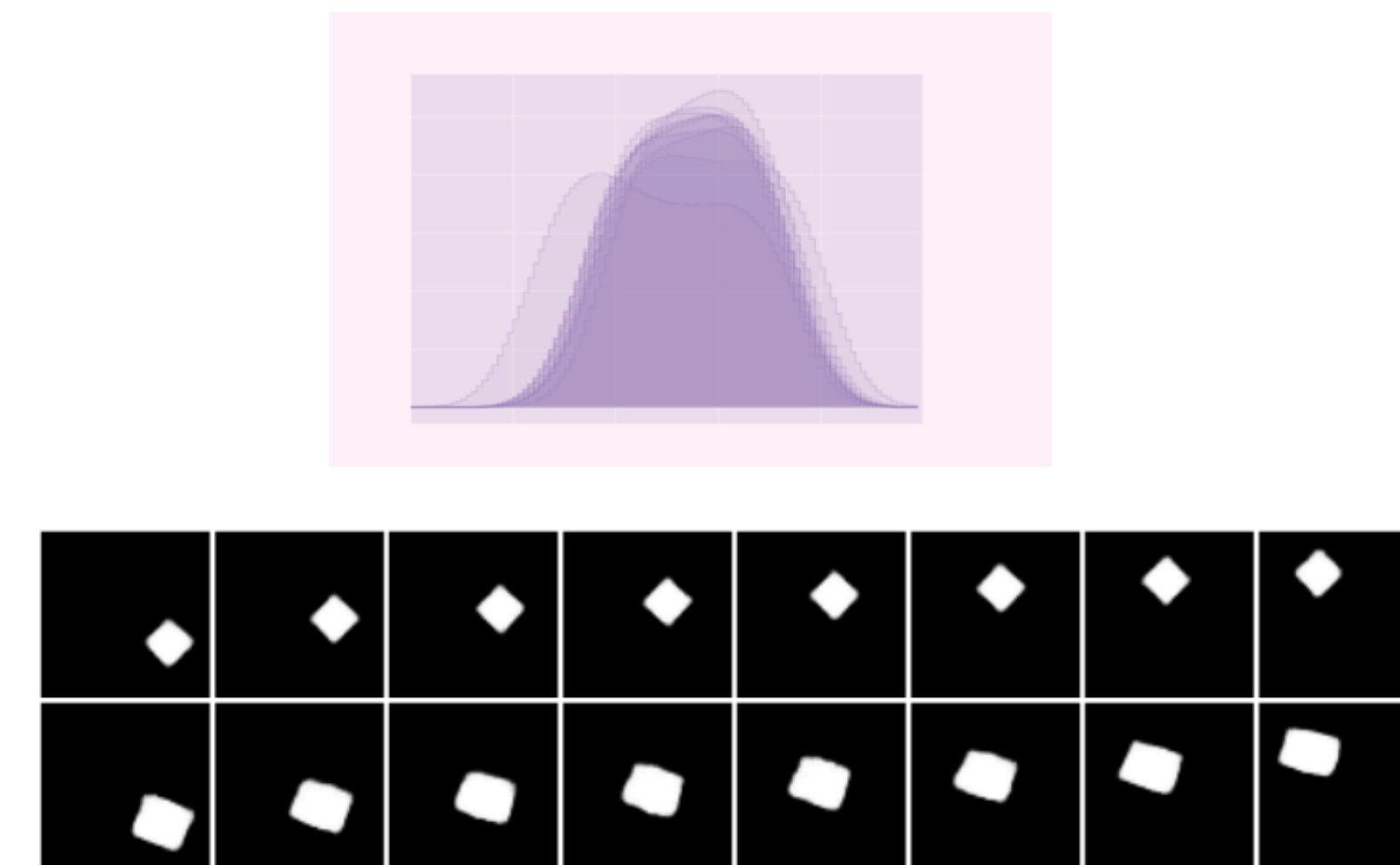
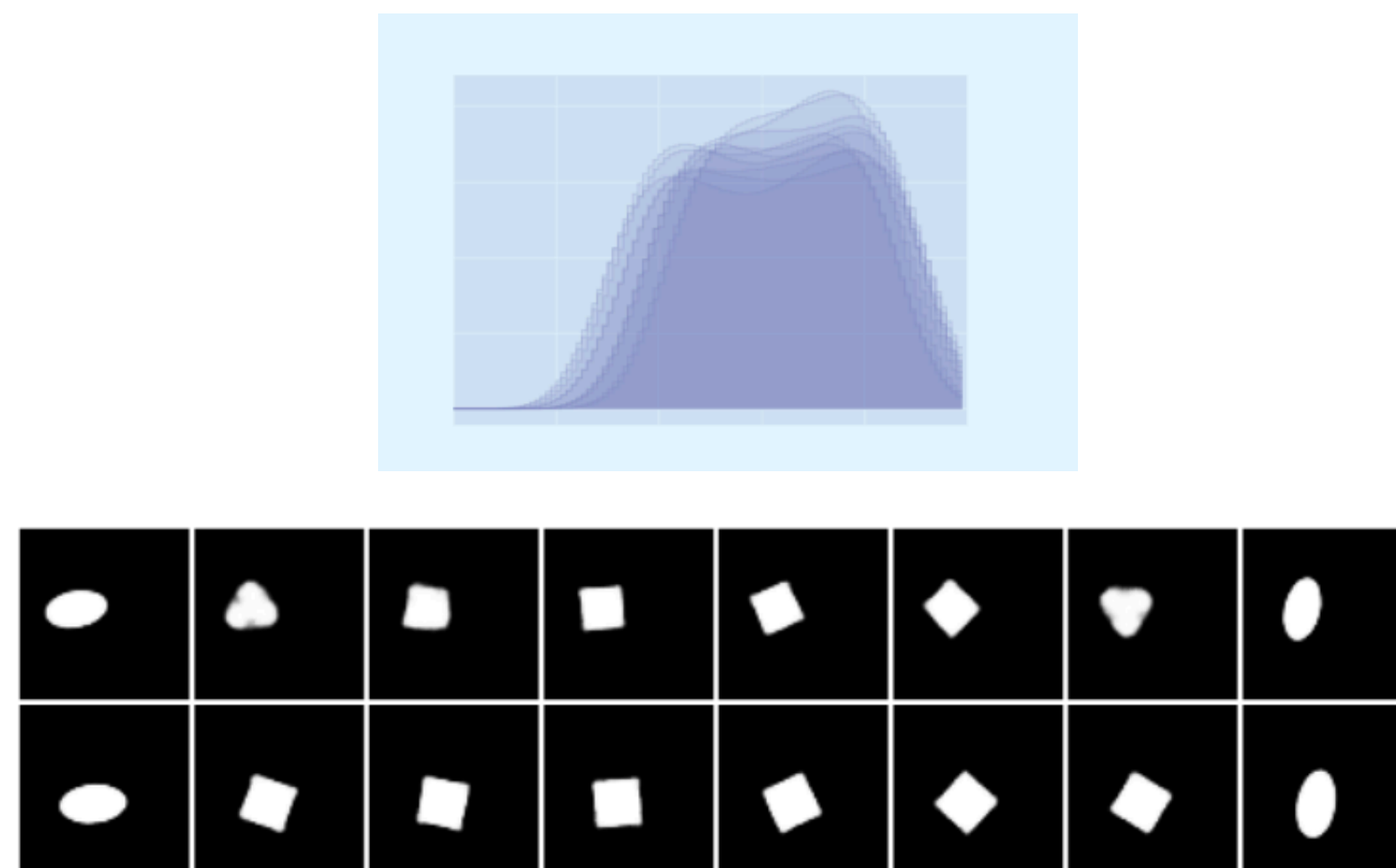
$\mathcal{M}_{\text{model}}$

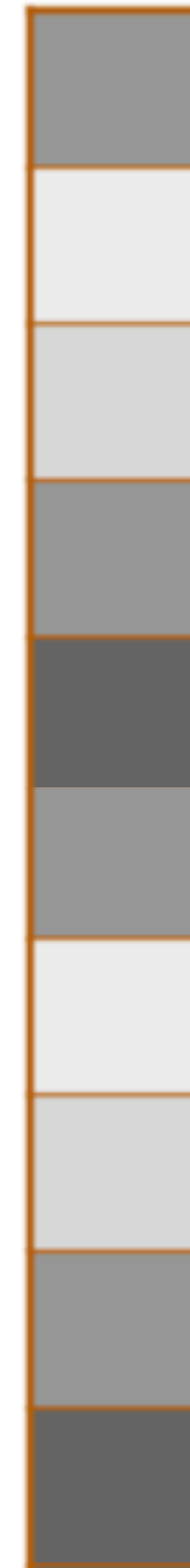






$\mathcal{M}_{\text{model}}$





Glasses

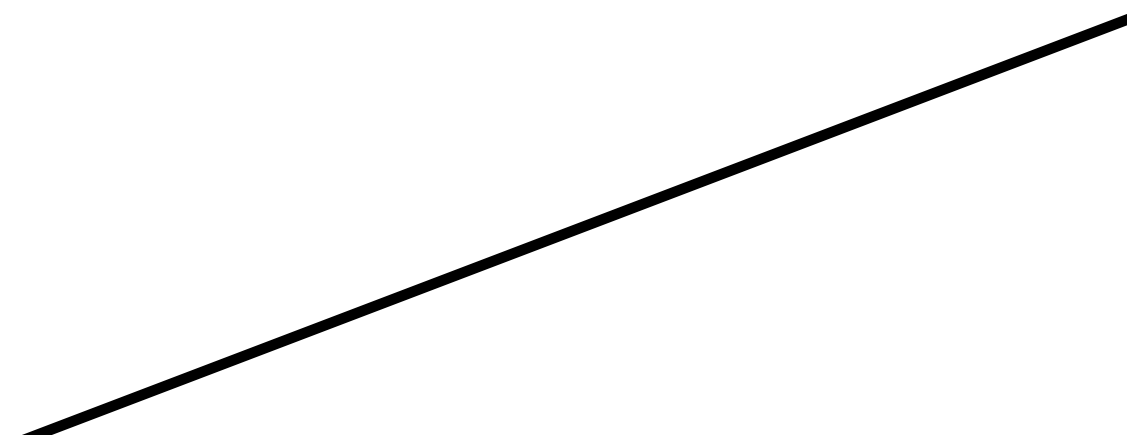
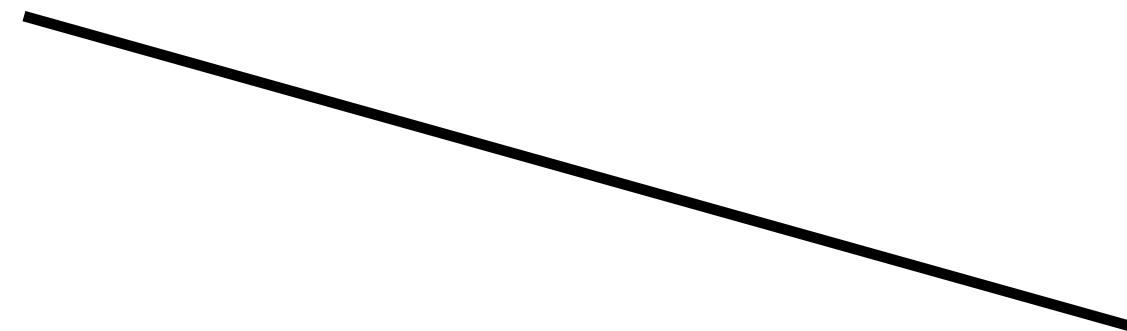
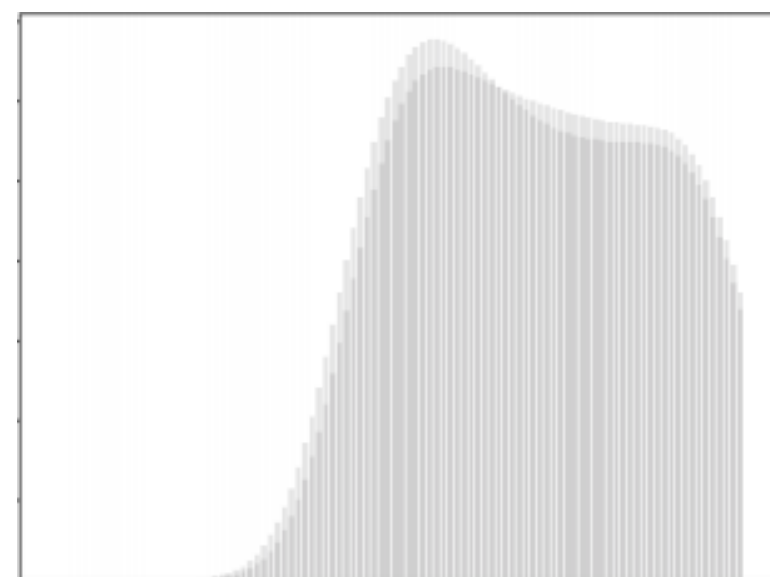
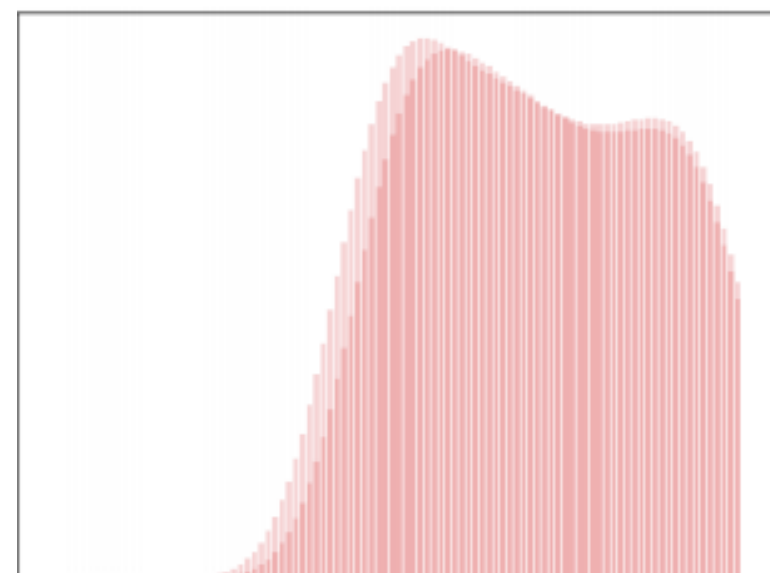
Beard

Hair

Eyes

Age





Glasses

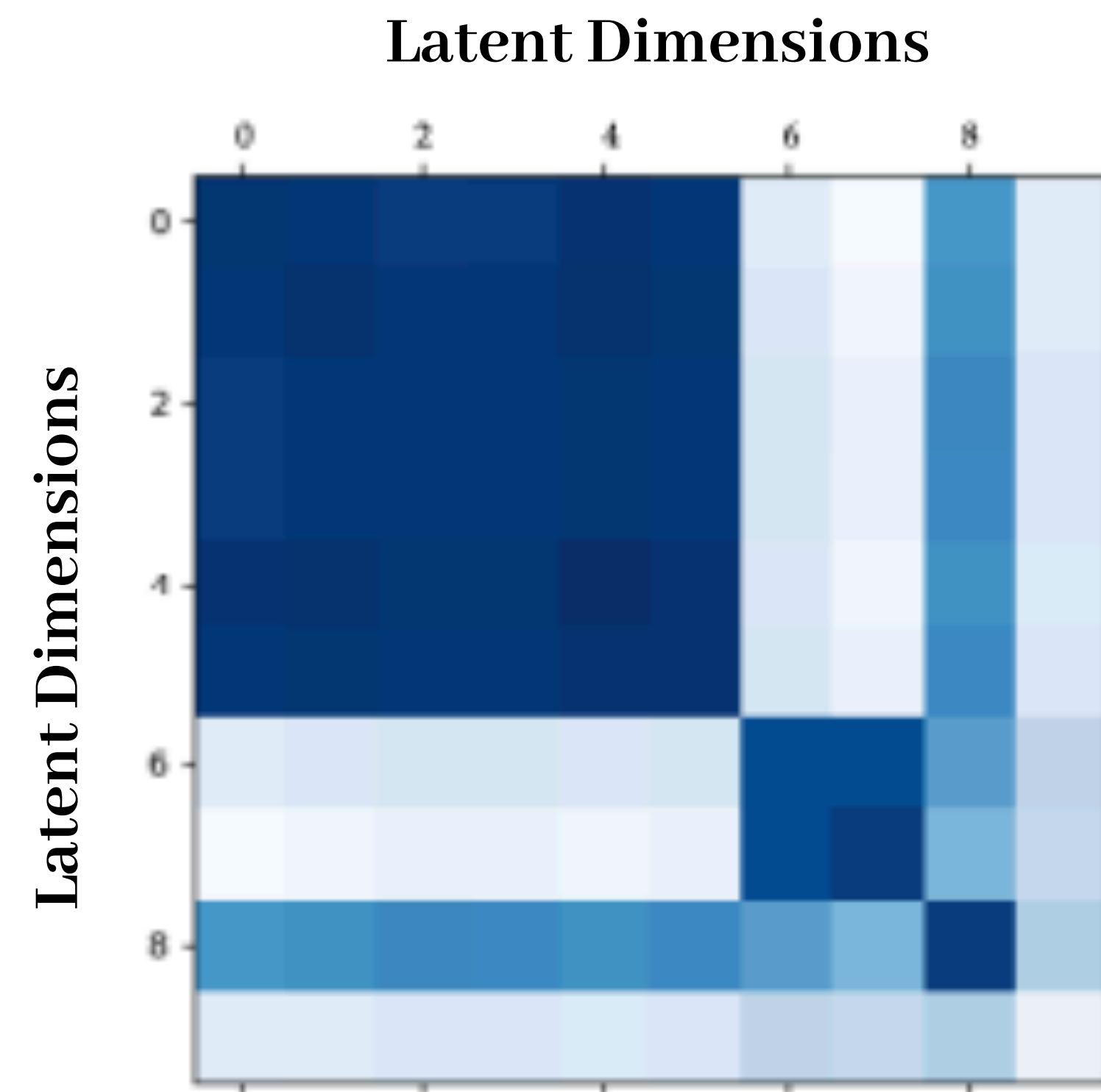
Beard

Hair

Eyes

Age

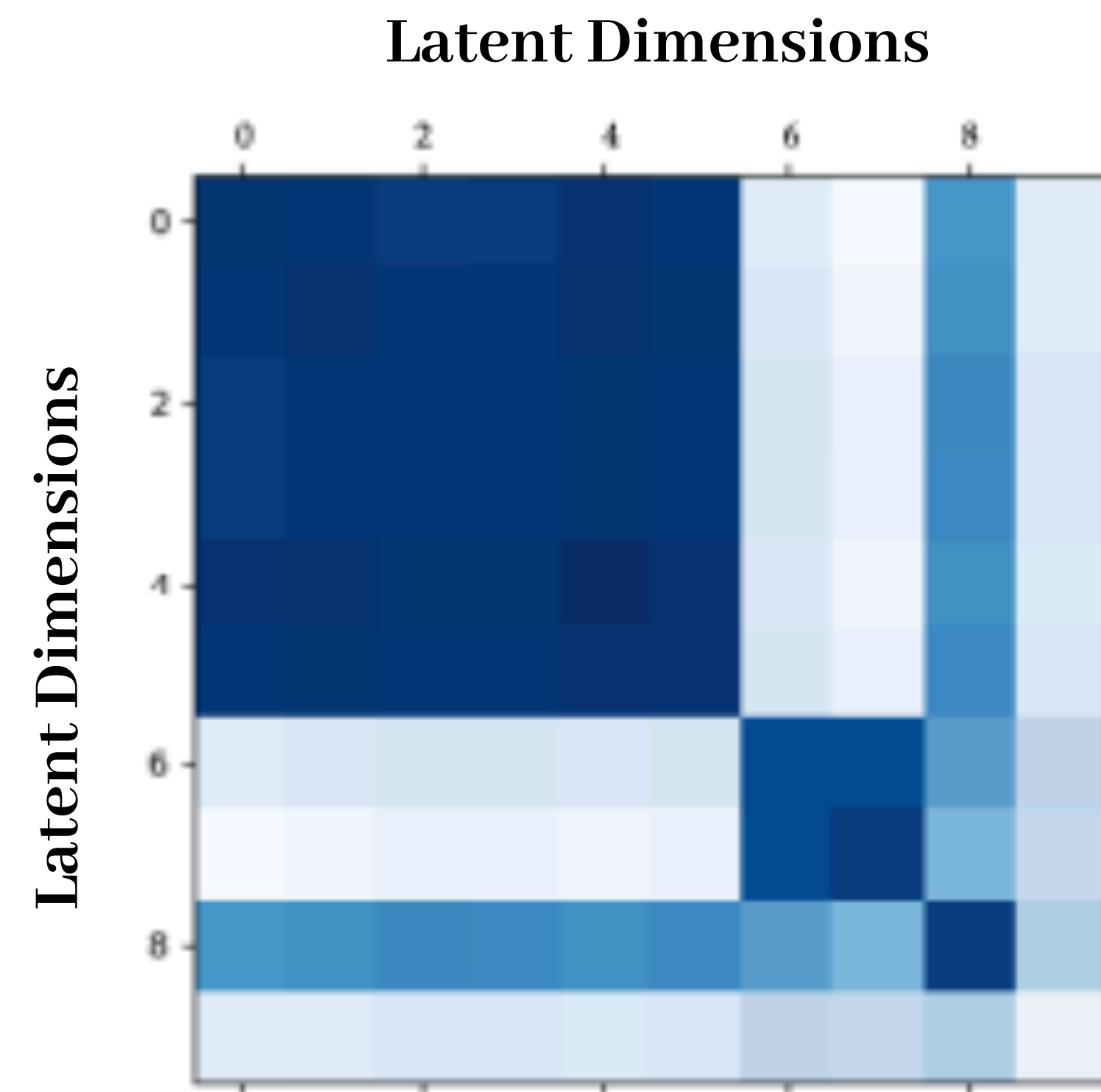
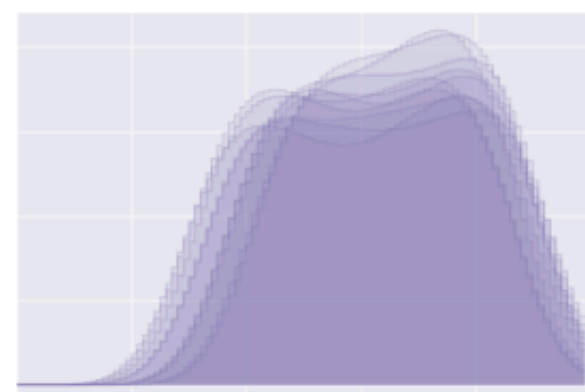
# dSprites $\beta$ -VAE (B)





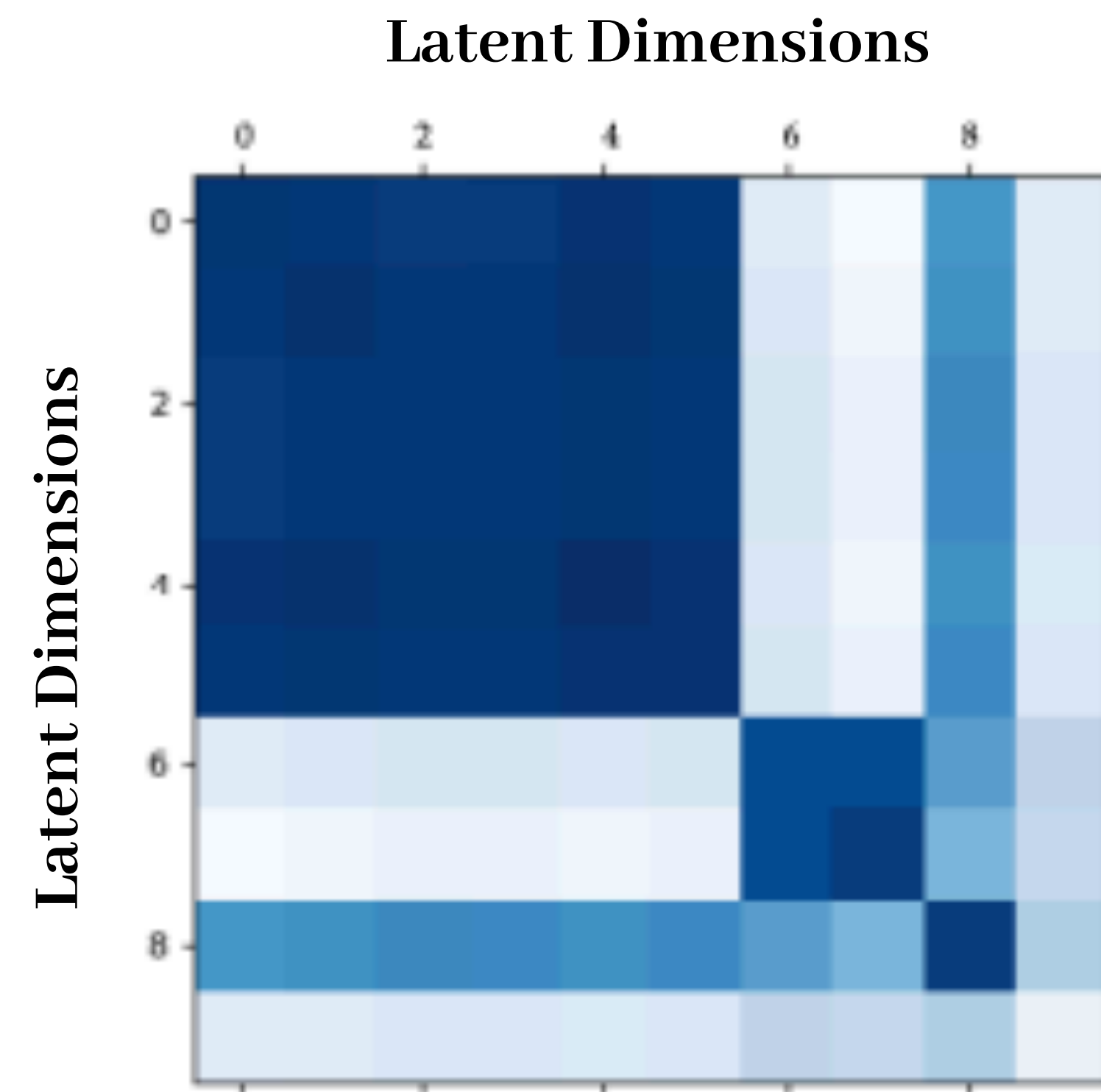
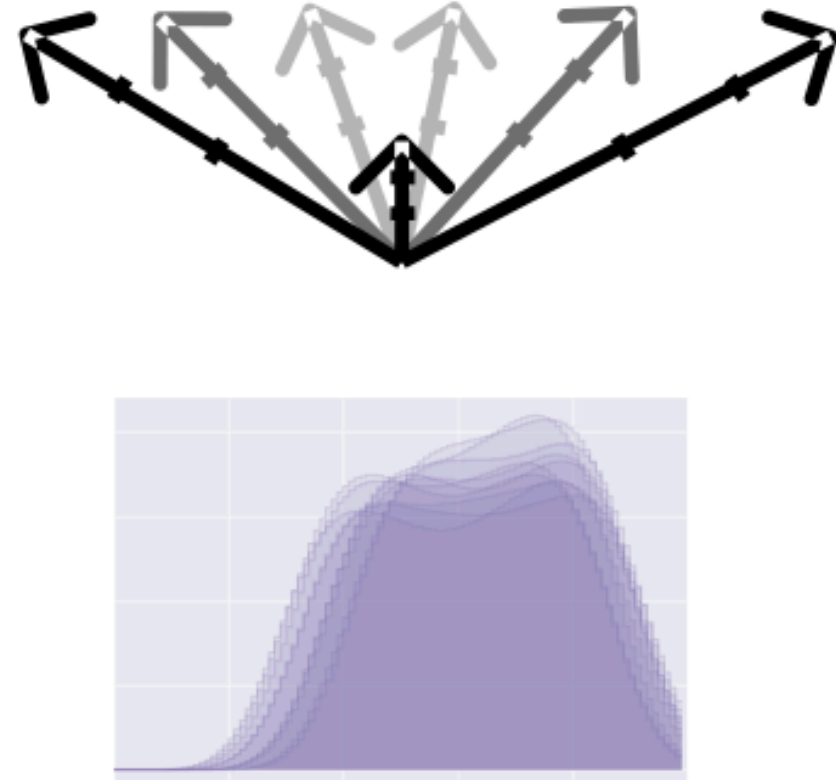
# dSprites $\beta$ -VAE (B)

Intracuster  
Inside Diagonal Clusters

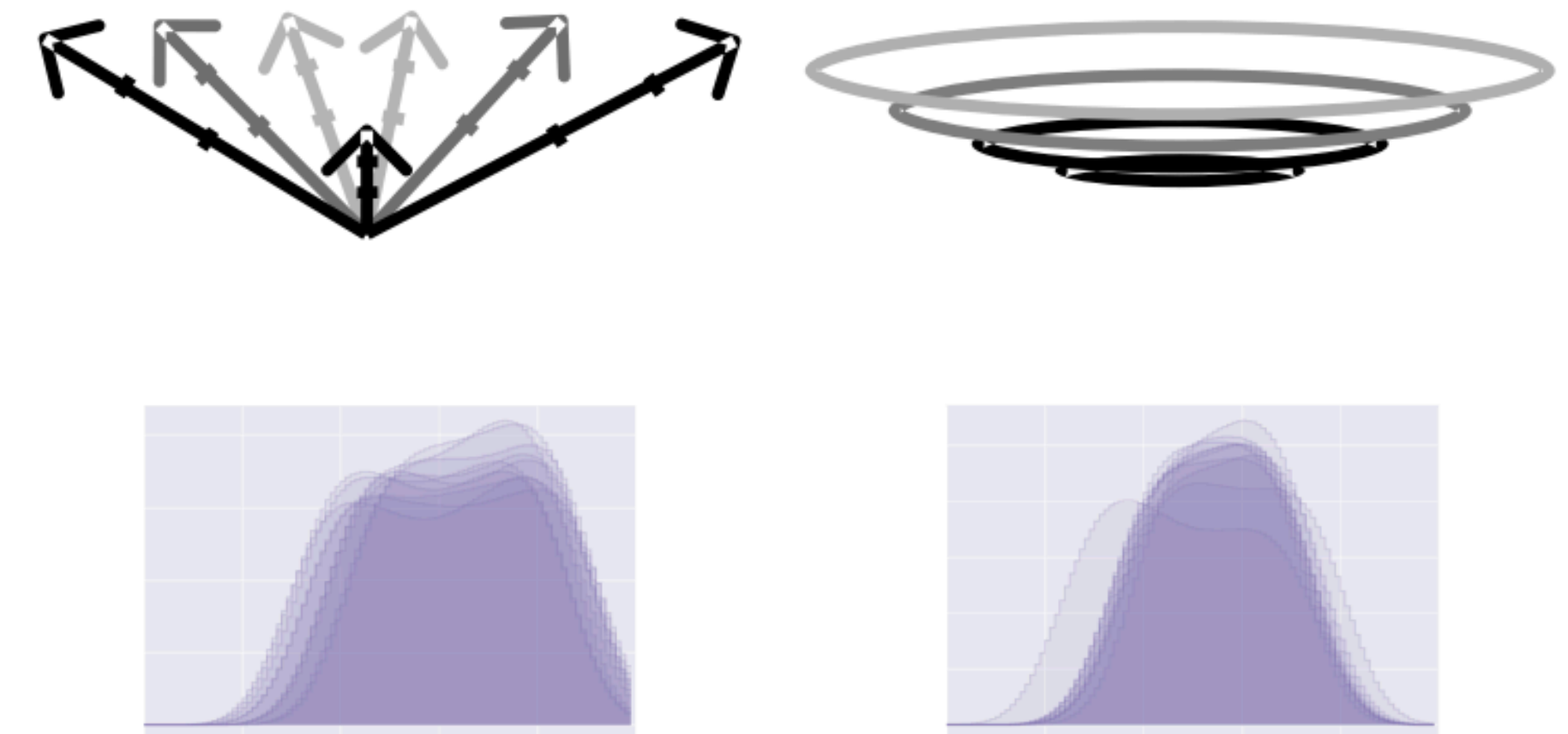


# dSprites $\beta$ -VAE (B)

Intracuster  
Inside Diagonal Clusters



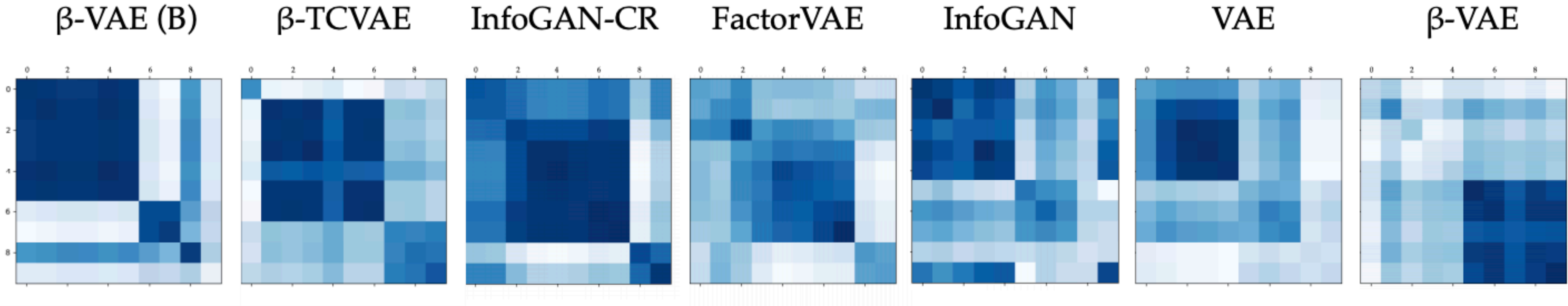
Extracuster  
Outside Diagonal Clusters



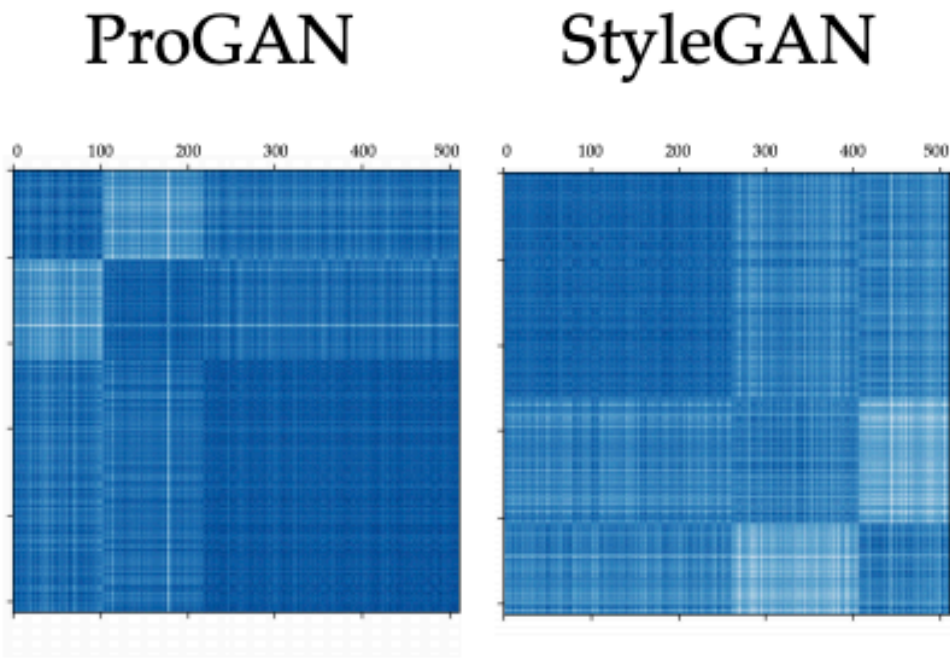


Spectrally Coclustered Topological Similarity Matrices

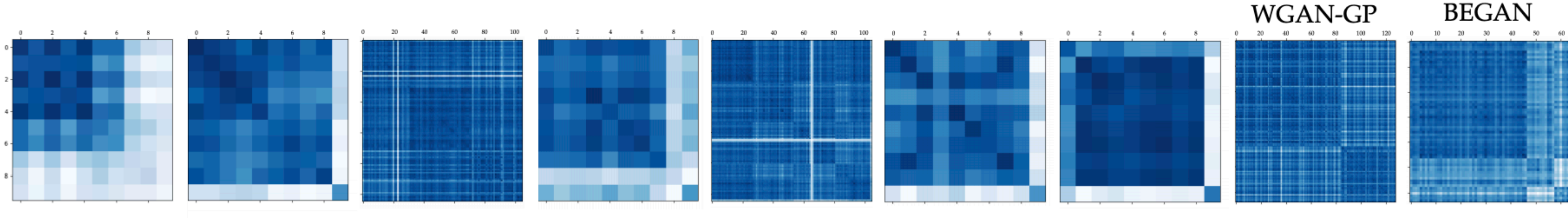
dSprites



Celeba-HQ



Celeba



## Prior methods

1. Depends on the **architecture** specifically with an **external model**, e.g. encoder and/or classifier
2. **Supervision** is required for a classifier
3. Tuned to a **specific dataset**, e.g. custom preprocessing on face images

## Ours

1. Uses an **intrinsic property** of a generative model, without reliance on external models or custom architectures
2. **Unsupervised** and supervised variants both available
3. Procedure can be **applied across datasets** — and architectures, as above