PopulAtion Parameter Averaging (PAPA)

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Structure

1. Leveraging a population of models
   - Ensembling
   - Weight averaging
2. PopulAtion Parameter Averaging (PAPA)
   - PAPA-all: occasionally replacing the weights by the average
   - PAPA: slowly pushing the weights toward the average
   - Why does averaging the weights of a population helps?
3. Results
4. Conclusion

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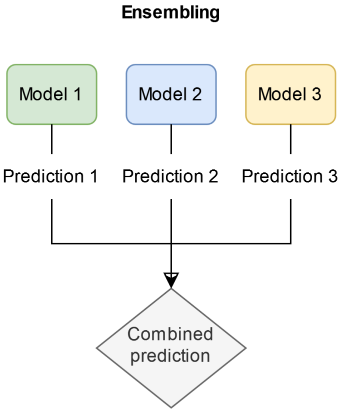

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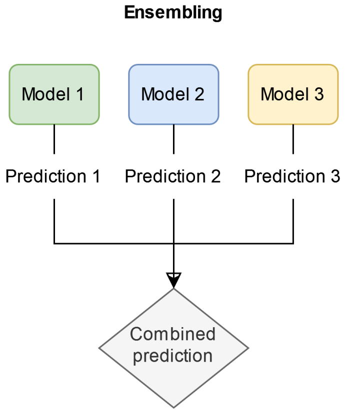

Ensembling: Combine the predictions of multiple models for a **massive boost in performance**

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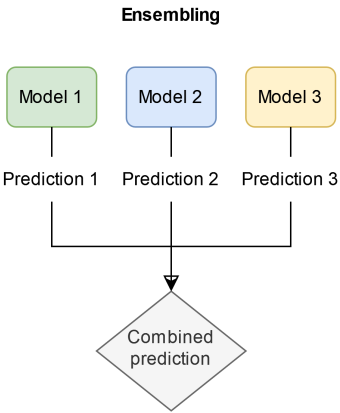

Ensembling: Combine the predictions of multiple models for a **massive boost in performance**

Simplest form of ensembling: averaging the prediction of all models

The curse of ensembling

Although powerful ensembling has drawbacks:

1. need to train multiple models
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Solution: weight averaging

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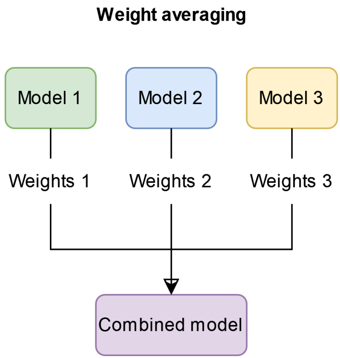

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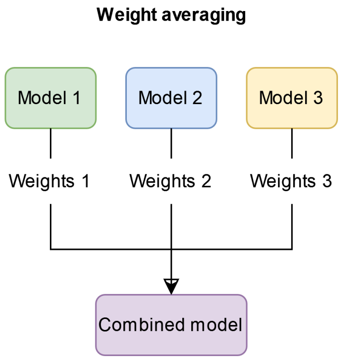

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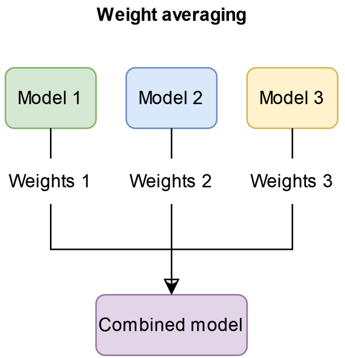

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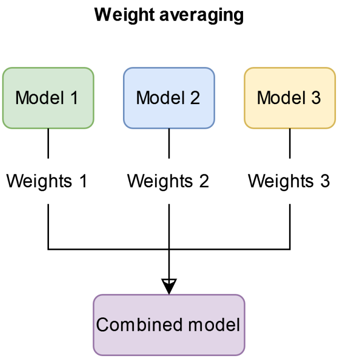

Averaging weights: Combine multiple models into one
Problem: Averaging does not work well in most scenarios
Question: Can we find a way to make it work well?

Permutation-alignment methods for better averaging

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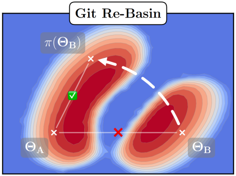

Figure 1: **Git Re-Basin merges models by teleporting solutions into a single basin.** Θ_B is permuted into functionally-equivalent $\pi(\Theta_B)$ so that it lies in the same basin as Θ_A.

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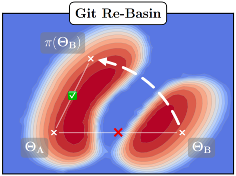

Figure 1: **Git Re-Basin merges models by teleporting solutions into a single basin.** Θ_B is permuted into functionally-equivalent $\pi(\Theta_B)$ so that it lies in the same basin as Θ_A.

Although powerful, permutation-alignment is not enough when averaging ≥ 3 models.

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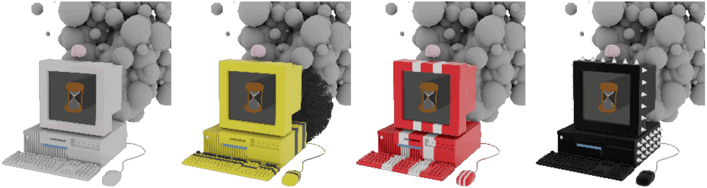
This is the key behind PAPA! We ensure during training that weights do not grow too dissimilar over time.

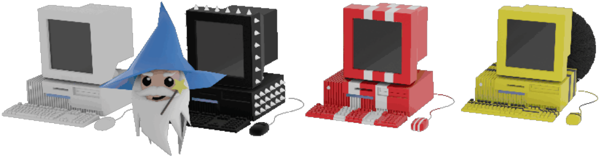
Overview

PAPA-all



4 networks (from 4 different random initializations) are trained independently on different data orderings, regularizations, and data-augmentations; each network learns slightly different features.

PAPA-all

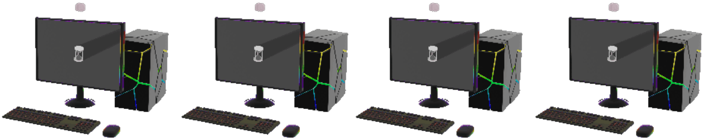


After K epochs, the 4 networks are combined through averaging to create a single averaged network that contains the features of each network and performs significantly better.

PAPA-all

The averaged network is duplicated to form the new population.

PAPA-all



The networks are trained independently again on different data ordering, regularizations, and data-augmentations.

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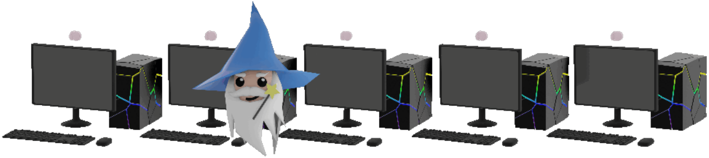

After k epochs, the networks are averaged again (and the process is repeated every k epochs until the end of training).

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By averaging occasionally, we gain in performance from averaging, but prevent misalignment due to dissimilar weights!

Model soups

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Greedy soup: Greedy average of "meaningful" models

- Sort models in order of decreasing train (or valid) accuracy
- Soup = weights of the first model
- (For loop) Try adding the next model to the soup (averaging all models weights equally); if train accuracy improves, add it to the soup

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Rescales the convolution/linear layers so that:

$$\mathbb{E}[X_\alpha] = (1-\alpha) \cdot \mathbb{E}[X_1] + \alpha \cdot \mathbb{E}[X_2], \tag{1}$$
$$\mathrm{std}(X_\alpha) = (1-\alpha) \cdot \mathrm{std}(X_1) + \alpha \cdot \mathrm{std}(X_2). \tag{2}$$

where X_α, X_1, X_2 are the features of the interpolated, first, and second networks.

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REPAIR makes the averaged network slightly **more performant** in PAPA

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Solution: At every SGD step, interpolate between current weights and population average:

$$\theta_j \leftarrow \alpha_{papa}\theta_j + (1 - \alpha_{papa})\bar{\theta},$$

where $\alpha_{papa} = 0.999$, θ_j is the weight of the j-th network, and $\bar{\theta}$ is the population average of the weights.

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We can also interpret PAPA as amortizing $\alpha_{papa} = 0.999$ on many steps.
Example: CIFAR-10, averaging every 5 epochs, mini-batch of 64
$\implies \alpha_{papa} = .999^{(781 \text{ iterations} \times 5 \text{ epochs})} \approx 0.02$.

Visualizing the effect of PAPA variants

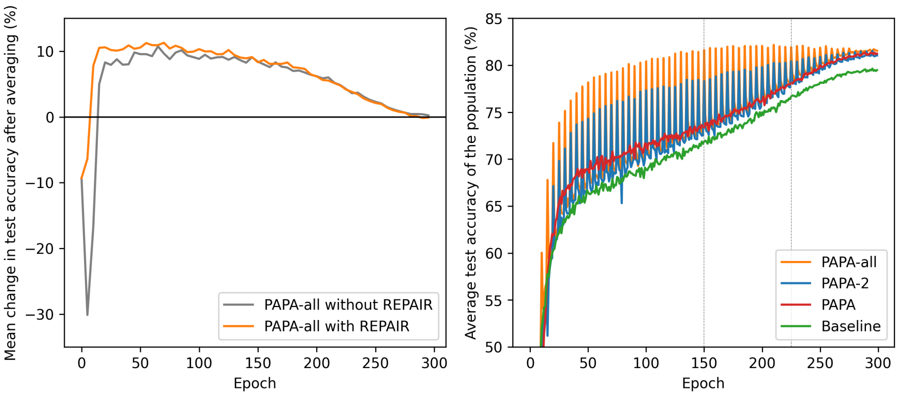

Figure: PAPA variants on CIFAR-100 when averaging every 5 epochs

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We demonstrate that:

- PAPA models share most of their features (proving feature mixing)
- Performance gain immediately after averaging (proving a benefit from feature mixing)

Thus, surprisingly, averaging weights also leads to averaging features in deep neural networks. More evidence exists in the permutation-alignment literature [Ainsworth et al., 2022].

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Table 1: Test accuracy from ensembles and soups with varying data augmentations and regularization

	Baseline		PAPA		PAPA-all		PAPA-2	
	Ensemble	GreedySoup	Ensemble	AvgSoup	Ensemble	AvgSoup	Ensemble	AvgSoup
# of models	p	1	p	1	p	1	p	1
CIFAR-10 ($n_{epochs} = 300, p = 10$)								
VGG-11	**95.2** (0.1)	94.0 (0.1)	94.9 (0.1)	**94.8** (0.0)	94.1 (0.2)	94.1 (0.2)	94.5 (0.1)	94.4 (0.1)
ResNet-18	**97.5** (0.0)	96.8 (0.2)	97.4 (0.1)	**97.4** (0.1)	97.3 (0.1)	97.3 (0.1)	97.1 (0.0)	97.1 (0.1)
CIFAR-100 ($n_{epochs} = 300, p = 10$)								
VGG-16	**82.2** (0.1)	77.8 (0.1)	79.6 (0.4)	**79.4** (0.3)	79.0 (0.4)	78.9 (0.4)	79.0 (0.3)	78.9 (0.3)
ResNet-18	**84.3** (0.3)	80.2 (0.6)	82.2 (0.1)	**82.1** (0.2)	81.8 (0.0)	81.8 (0.0)	81.3 (0.3)	81.2 (0.3)
Imagenet ($n_{epochs} = 90, p = 3$)								
ResNet-50	**78.7**	76.8	78.4	**78.4**	77.7	77.7	77.8	77.8
Fine-tuning on CIFAR-100 ($n_{epochs} = 50$, $p = 2, 4, 5$ respectively for EfficientNetV2, EVA-02, ConViT)								
EffNetV2-S	**91.7** (0.3)	91.3 (0.4)	91.6 (0.3)	**91.4** (0.5)	91.4 (0.4)	91.1 (0.4)	91.3 (0.6)	91.3 (0.6)
EVA-02-Ti	90.6 (0.1)	90.4 (0.1)	**90.7** (0.3)	90.6 (0.2)	**90.7** (0.6)	**90.7** (0.5)	90.5 (0.3)	90.4 (0.3)
ConViT-Ti	**88.8** (0.2)	87.9 (0.2)	88.6 (0.2)	**88.4** (0.2)	88.2 (0.2)	88.1 (0.1)	88.2 (0.2)	88.2 (0.3)

Comparing PAPA to single models trained longer

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Baseline	PAPA	PAPA-all	PAPA-2
Mean	AvgSoup	AvgSoup	AvgSoup
VGG-16: No data augmentations or regularization			
74.15 (0.1)	**76.04**	75.13	75.10
VGG-16: With random data augmentations			
77.44 (0.1)	**79.36** (0.3)	78.89 (0.4)	78.91 (0.3)
ResNet-18: No data augmentations or regularization			
78.23 (0.6)	78.11	**78.59**	77.90
ResNet-18: With random data augmentations			
79.88 (0.5)	82.06 (0.2)	81.77 (0.0)	81.23 (0.3)

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Conclusion: PAPA is an **efficient way of parallelizing training length** over multiple networks

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Future directions:

1. PAPA for large generative and language models
2. PAPA for continual learning
3. Better understand features propagation between networks in PAPA
4. Different weighting schemes to emphasize generalizable features

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