Theoretically Upper-Bounding the Expected Adversarial Robustness of GNNs.

Yassine Abbahaddou [1], Sofiane Ennadir [2]

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May 17, 2024

Today we present work that was done under the supervision of

Prof. Johannes Lutzeyer
Assistant Professor LIX

Prof. Henrik Boström
Professor KTH

Prof. Michalis Vazirgiannis
Distinguished Professor LIX

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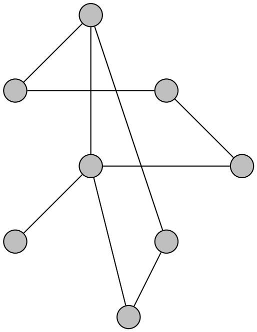

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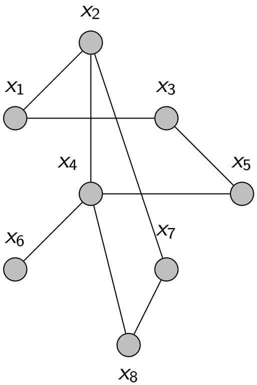

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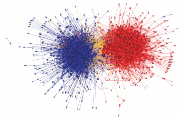

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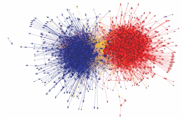

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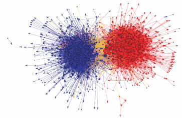

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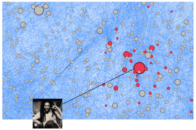

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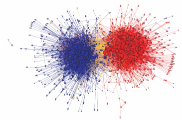

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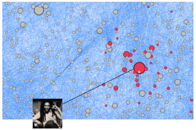

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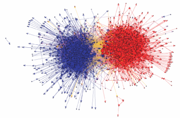

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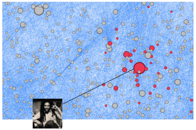

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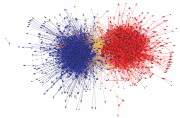

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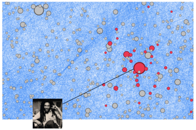

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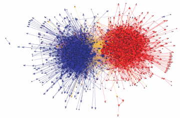

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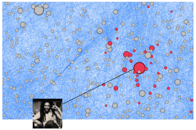

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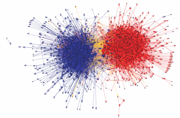

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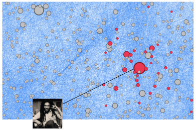

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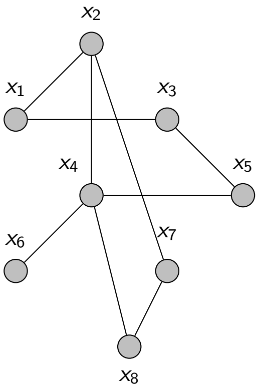

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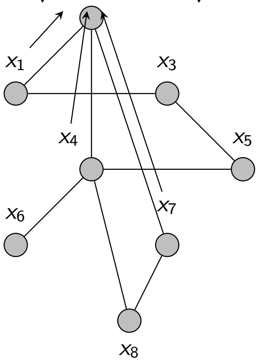
E.g., the Graph Convolutional Network (GCN, Kipf and Welling, 2017)

$$\tilde{A}X.$$

$$m_2^{(1)} = \frac{1}{\sqrt{d_2}} \sum_{i\in\{1,4,7\}} \frac{x_i}{\sqrt{d_i}}$$



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H^{(1)} = \mathrm{ReLU}\left(\tilde{A}XW^{(1)}\right).
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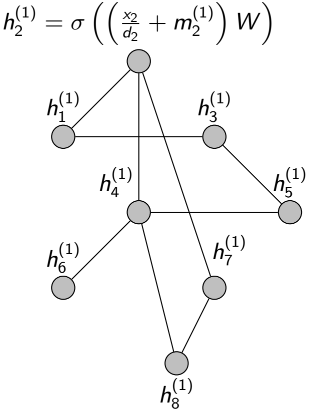

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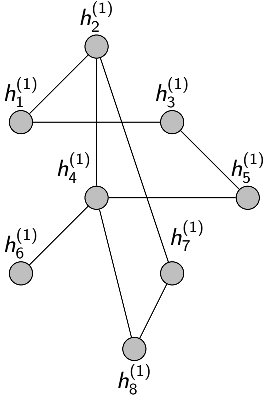

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Iteratively performing the message-passing and update computations allows us to build 'deep' learning models, e.g., a 3-layer GCN

$$\hat{y} = \sigma \left(\tilde{A} \, \mathrm{ReLU} \left(\tilde{A} \, \mathrm{ReLU} \left(\tilde{A} X W^{(1)} \right) W^{(2)} \right) W^{(3)} \right).$$

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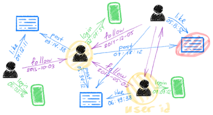

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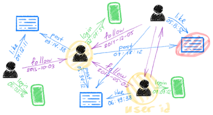

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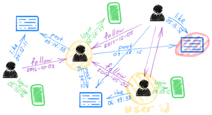

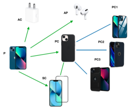

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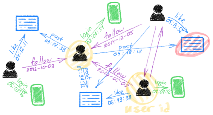

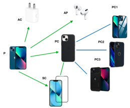

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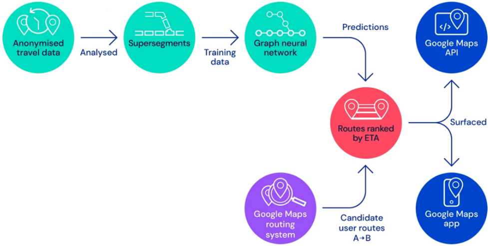

Traffic prediction with advanced Graph Neural Networks - DeepMind

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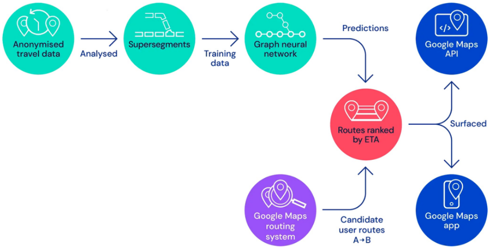

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By: HT CORRESPONDENT | Updated on: Aug 20 2022, 19:09 IST

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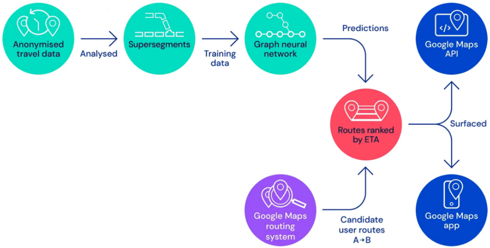

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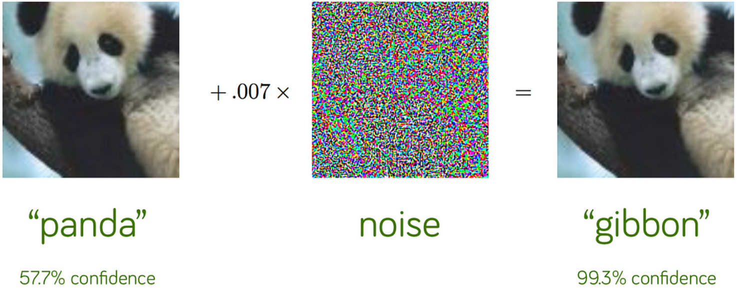

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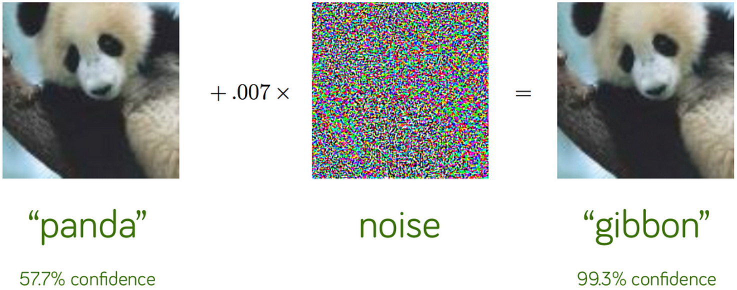

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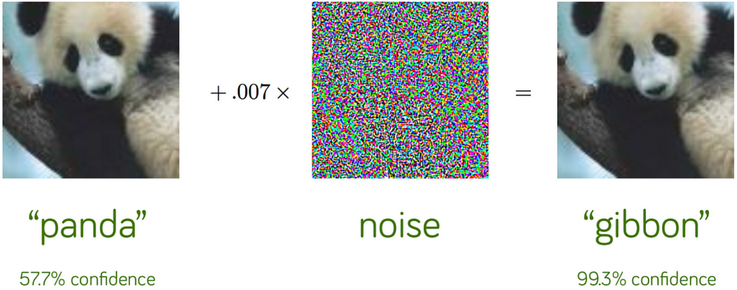

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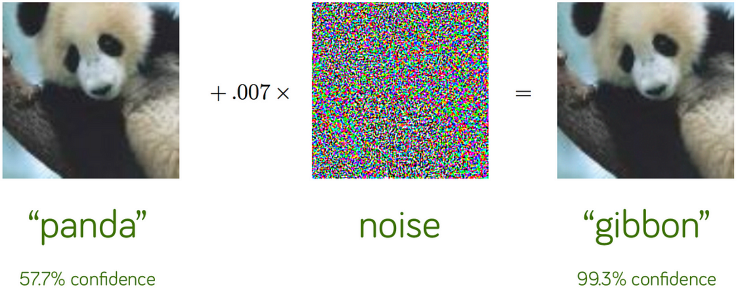

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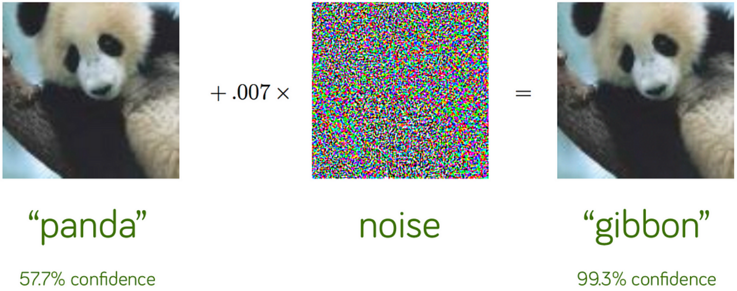

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The set of adversarial graphs can be written as:

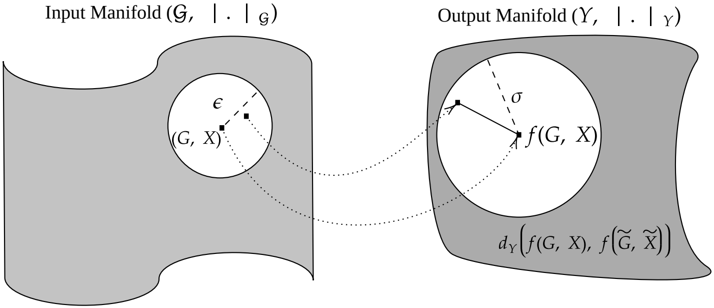
$$\hat{\mathcal{G}} = \{ [\tilde{G}, \tilde{X}] \mid d^{\alpha,\beta}([G, X], [\tilde{G}, \tilde{X}]) \leq \epsilon : f([\tilde{G}, \tilde{X}]) \neq f([G, X]) \}$$

Graph Adversarial Attacks

We introduce the concept of **"Adversarial Risk"** for a graph-based classifier f as follows:

$$\mathbf{Adv}_{\epsilon}^{\alpha,\beta}[\mathbf{f}] = \mathbb{P}_{(\mathbf{G},\mathbf{X})\sim\mathcal{D}_{\mathcal{G},\mathcal{X}}}[(\tilde{\mathbf{G}},\tilde{\mathbf{X}}) \in \mathbf{B}^{\alpha,\beta}(\mathbf{G},\mathbf{X},\epsilon) : \mathbf{d}_{\mathcal{Y}}(\mathbf{f}(\tilde{\mathbf{G}},\tilde{\mathbf{X}}),\mathbf{f}(\mathbf{G},\mathbf{X})) > \sigma], \tag{1}$$

with: $B^{\alpha,\beta}(G,X,\epsilon) = \{(\tilde{G},\tilde{X}) : d^{\alpha,\beta}([G,X],[\tilde{G},\tilde{X}]) < \epsilon\}$ being the input's graph neighborhood.



Definition (Graph Adversarial Robustness).

The graph-based function $f : (\mathcal{A},\mathcal{X}) \rightarrow \mathcal{Y}$ is said to be $(\epsilon,\gamma)-$ robust if its **adversarial risk** is upper-bounded, i. e., $Adv_{\epsilon}^{\alpha,\beta}[f] \leq \gamma$ with respect to the chosen graph distances.

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Problem: Most defense approaches for GNNs defend structural attacks altering A. There exists very little work on how to defend against attacks on the node features X.

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Main Theorem (Upper Bound on GCN Vulnerability).

We consider node-feature attacks on the input graph (A, X), with a budget ϵ and L-layer GCNs with weight matrices $W^{(i)}$ for $i \in \{1, \ldots, L\}$.
Then, the **adversarial risk** of GCNs is upper bounded by

$$\gamma = \prod_{i=1}^{L} \|W^{(i)}\|_1 \frac{\epsilon \sum_{u \in \mathcal{V}} \hat{w}_u}{\sigma},$$

with $\hat{w}_u$ denoting the sum of normalized walks of length $(L-1)$ starting from node u.

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Insight: Our computed upper bound on the adversarial risk of a GCN is **depedent on the weight norm**. Specifically, **smaller** $\prod_{i=1}^{L} \|W^{(i)}\|_1$ yields a **more robust GCN.**

Generalization of the Theoritical Results

Recall, Graph Neural Networks (GNNs) take both a graph A and node features X as input.

Theorem 2 (Structural Attacks).

We consider structural attacks on the input graph (A, X), with a budget ϵ and L-layer GCNs with weight matrices $W^{(i)}$ for $i \in \{1, \ldots, L\}$.
Then, the **adversarial risk** of GCNs is upper bounded by

$$\gamma = \prod_{i=1}^{L} \|W^{(i)}\|_2 \|X\|_2 \epsilon (1 + L \prod_{i=1}^{L} \|W^{(i)}\|_2)/\sigma.$$

Insight: The computed upper bound in the case of structural case shows similar findings as the case of node-features based attacks. Specifically, the bound is **depedent on the weight norm**.

Methodology

Fact: Orthonormal matrices have norm 1.

$\Rightarrow$ According to our bound; a GNN with orthonormal weight matrices should be more robust.

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Björk Orthonormalisation Algorithm (A. Björck and C. Bowie., 1971)

Given a weight matrix W we iteratively alter it to approximate the closest orthonormal matrix $\hat{W}$. When $\hat{W}_0 = W$, we recursively compute

$$\hat{W}_{k+1} = \hat{W}_k \left(I + \frac{1}{2} \left(I - \hat{W}_k^T \hat{W}_k \right) + \ldots + (-1)^p \binom{-1/2}{p} \left(I - \hat{W}_k^T \hat{W}_k \right)^p \right).$$

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Proposed Solution: In our *GCORN* model we propose the inclusion of several Björk Orthonormalisation iterations in each forward pass during the training of a GCN, **yielding weight matrices that approach orthonormality and thereby a more robust GNN**.

Estimation of Our Robustness Measure

- Goal: Empirically estimate $\mathbf{Adv}_{\epsilon}^{\alpha,\beta}[\mathbf{f}]$

$$Adv_{\epsilon}^{\alpha,\beta}[f] = \mathbb{E}_{\substack{(G,X)\sim\mathcal{D}_{\mathcal{G},\mathcal{X}},\\ (\tilde{G},\tilde{X})\in B^{\alpha,\beta}((G,X),\epsilon)}} \left[\mathbf{1}\{d_{\mathcal{Y}}(f(\tilde{G},\tilde{X}), f(G,X)) > \sigma\}\right].$$

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- Insight: Use Stratified Sampling
  - Sampling $\tilde{X}$ is equivalent to first sample $Z \in \mathbb{R}^{n\times K}$ from $\mathcal{B}_{\epsilon} = \left\{Z \in \mathbb{R}^{n\times K} : \|Z\|_{\mathcal{X}} \leq \epsilon\right\}$ and then set $\tilde{X} = X + Z$
  - Decomposition of $\mathcal{B}_{\epsilon}$

$$S_r = \{Z \in \mathbb{R}^{n\times K} : \|Z\|_{\mathcal{X}} = r\}, \qquad \mathcal{B}_{\epsilon} = \cup_{r\leq\epsilon} S_r; \qquad \forall r \neq r' \quad S_r \cap S_{r'} = \emptyset.$$

Lemma

Let $\mathbb{R}^K$ be the real finite-dimensional space and ϵ a positive real number. If $R^{(p)}$ is the random variable indicating the maximum of the L_p norm's values inside the ball of radius ϵ, i.e., $\mathcal{B}_{\epsilon} = \left\{Z \in \mathbb{R}^{n\times K} : \max_{i\in\{1,\dots,n\}}\|Z_i\|_p \leq \epsilon\right\}$. Then, for every $p > 0$, the density distribution of $R^{(p)}$ does not depends on p and is defined as follows, $p_{\epsilon}(r) = K\frac{1}{\epsilon}\left(\frac{r}{\epsilon}\right)^{K-1}\mathbf{1}\{0 \leq r \leq \epsilon\}$.

Estimation of Our Robustness Measure

- **Goal: empirically estimate $\mathbf{Adv}_{\epsilon}^{\alpha,\beta}[\mathbf{f}]$**

$$Adv_{\epsilon}^{\alpha,\beta}[f] = \mathbb{E}_{\substack{(G,X)\sim\mathcal{D}_{\mathcal{G},\mathcal{X}},\\ (\tilde{G},\tilde{X})\in B^{\alpha,\beta}((G,X),\epsilon)}} \left[\mathbf{1}\{d_{\mathcal{Y}}(f(\tilde{G},\tilde{X}), f(G,X)) > \sigma\}\right].$$

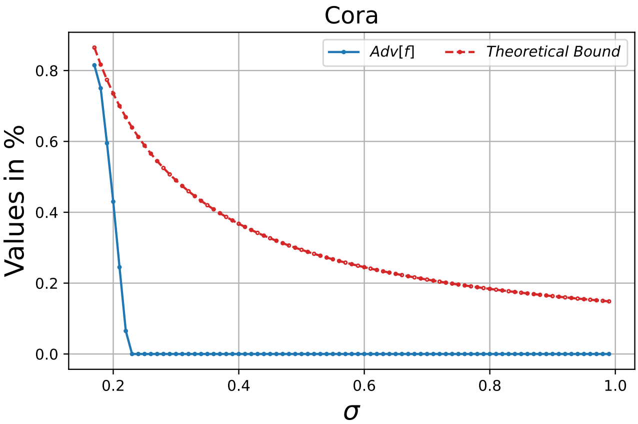
Algorithm Estimation of $Adv_{\epsilon}^{\alpha,\beta}[f]$.

Inputs: Sphere Radius : $\epsilon > 0$, Number of Samples L_{max}, Number of Input Graphs $|\mathcal{D}|$;
Initialize $Adv = 0$;
foreach $[G_i, X_i] \in \mathcal{D}$ **do**
  Initialize $Adv_i = 0$;
  foreach $l = 1, \dots, L_{max}$ **do**
    1. Sample a distance $r \in [0, \epsilon]$ from the prior distribution p_{ϵ};
    2. Uniformly sample $Z_l \in \mathbb{R}^{n\times K}$ from S_r;
    3. Choose $\tilde{X}_l = X_i + Z_l$;
    4. Update
       $Adv_i \leftarrow Adv_i + \mathbf{1}\{d_{\mathcal{Y}}(f(\tilde{G}_l, \tilde{X}_l), f(G,X)) > \sigma\}$
  end foreach
  $Adv_i = Adv_i/L_{max}$; $Adv = Adv + Adv_i$;
end foreach
Return $Adv/\mid\mathcal{D}\mid$

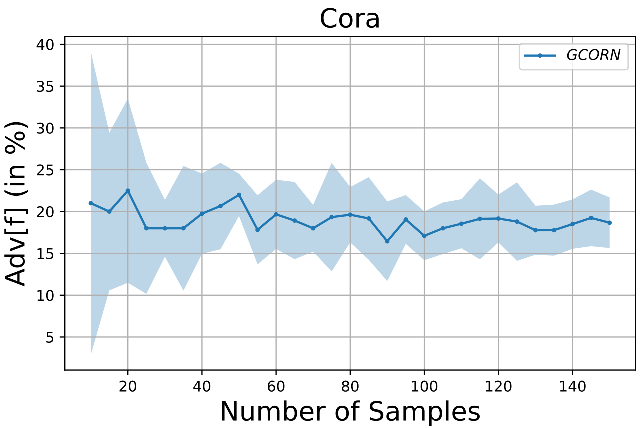
Tightness of the Computer Theoretical Upper-Bound

Robustness Inequality:

$$Adv_{\epsilon}^{\alpha,\beta}[f] \leq \gamma = \prod_{i=1}^{L} \|W^{(i)}\|_{\infty} \epsilon \hat{w}_G / \sigma$$



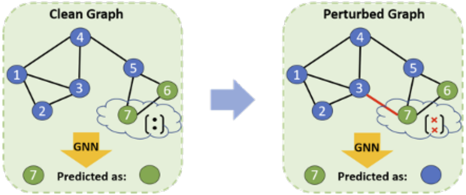
The Effect of Sampling on the Empirical Estimation Of $Adv_{\epsilon}^{\alpha,\beta}[f]$



Required Number of Samples based on the :

$$\frac{log(\alpha)}{log\left(1-\left(\frac{r}{\epsilon}\right)^{K}\right)}$$

Graph Adversarial Attacks



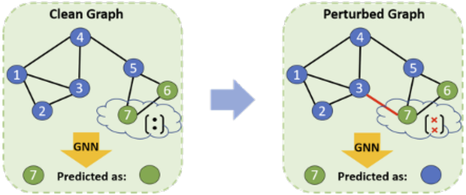
Different attack possibilities within the Graph:

- Edit Edges.
- Edit Nodes/Edges Features.
- Add/Delete Nodes.

And different settings:

- White Box (Full Knowledge).
- Black Box (No Knowledge assumed).

Graph Adversarial Attacks



Different attack possibilities within the Graph:

- Edit Edges.
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And different settings:

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Feature-based Attacks:

- Random Attack – Injecting noise from a scaled centered Gaussian $\mathcal{N}(0, 1)$.
- Gradient-based – Mainly using "PGD" and "Nettack".

Structure-based Attacks:

- Gradient-based – "Mettack" and "PGD".
- Probabilistic gradient method – based on "DICE".

Results

Table: Node classification accuracy (± standard deviation) for feature-based attacks.

Attack	Dataset	GCN	GCN-k	AirGNN	RGCN	ParsevalR	GCORN
Random ($\psi = 0.5$)	Cora	68.4 ± 1.9	69.2 ± 2.6	73.5 ± 1.9	71.6 ± 0.3	72.9 ± 0.9	**77.1 ± 1.8**
	CiteSeer	57.8 ± 1.5	62.3 ± 1.2	64.6 ± 1.6	63.7 ± 0.6	65.1 ± 0.8	**67.8 ± 1.4**
	PubMed	68.3 ± 1.2	71.2 ± 1.1	70.9 ± 1.3	71.4 ± 0.5	71.8 ± 0.8	**73.1 ± 1.1**
	CS	85.3 ± 1.1	86.7 ± 1.1	87.5 ± 1.6	88.2 ± 0.9	87.6 ± 0.6	**89.8 ± 1.2**
	OGBN-Arxiv	68.2 ± 1.5	52.8 ± 0.5	66.5 ± 1.3	63.8 ± 1.9	68.3 ± 1.9	**69.1 ± 1.8**
Random ($\psi = 1.0$)	Cora	41.7 ± 2.1	46.3 ± 2.8	53.7 ± 2.2	52.8 ± 1.6	55.3 ± 1.2	**57.6 ± 1.9**
	CiteSeer	38.2 ± 1.3	45.3 ± 1.4	49.8 ± 2.1	43.7 ± 2.2	51.2 ± 1.2	**57.3 ± 1.7**
	PubMed	60.1 ± 1.7	62.3 ± 1.3	62.4 ± 1.2	61.9 ± 1.2	61.3 ± 1.7	**65.8 ± 1.4**
	CS	69.9 ± 1.3	73.2 ± 0.9	76.7 ± 2.8	76.2 ± 1.4	78.7 ± 1.2	**81.3 ± 1.6**
	OGBN-Arxiv	66.4 ± 1.9	46.6 ± 0.6	62.7 ± 1.6	63.0 ± 2.4	66.1 ± 0.7	**67.3 ± 2.1**
PGD	Cora	54.1 ± 2.4	58.3 ± 1.6	68.2 ± 1.8	62.5 ± 1.2	68.6 ± 1.7	**71.1 ± 1.4**
	CiteSeer	52.3 ± 1.1	59.6 ± 1.6	59.3 ± 2.1	61.9 ± 1.1	62.1 ± 1.5	**65.6 ± 1.4**
	PubMed	66.1 ± 2.1	67.3 ± 1.3	70.8 ± 1.7	69.5 ± 0.9	68.9 ± 2.1	**72.3 ± 1.3**
	CS	71.3 ± 1.1	74.1 ± 0.8	76.3 ± 2.1	76.6 ± 1.2	77.3 ± 0.6	**79.6 ± 1.2**
	OGBN-Arxiv	67.5 ± 0.9	49.9 ± 0.7	55.7 ± 0.9	63.6 ± 0.7	67.6 ± 1.2	**68.1 ± 1.1**
Nettack	Cora	60.9 ± 2.5	64.2 ± 5.2	66.7 ± 3.8	63.4 ± 3.8	67.5 ± 2.5	**68.3 ± 1.4**
	CiteSeer	55.8 ± 1.4	71.7 ± 1.4	67.5 ± 2.5	70.8 ± 3.8	69.2 ± 3.8	**77.5 ± 2.5**
	PubMed	60.0 ± 2.5	65.8 ± 2.9	69.2 ± 1.4	**71.7 ± 3.8**	68.3 ± 1.4	70.8 ± 1.4
	CS	55.8 ± 1.4	71.6 ± 1.4	76.7 ± 1.4	71.7 ± 2.9	75.8 ± 2.8	**78.3 ± 1.4**
	OGBN-Arxiv	49.2 ± 2.9	53.3 ± 1.4	**56.7 ± 1.4**	52.6 ± 2.5	55.8 ± 1.4	55.8 ± 1.4

- Our **GCORN model often outperforms** existing defense approaches when subject to feature based attacks.

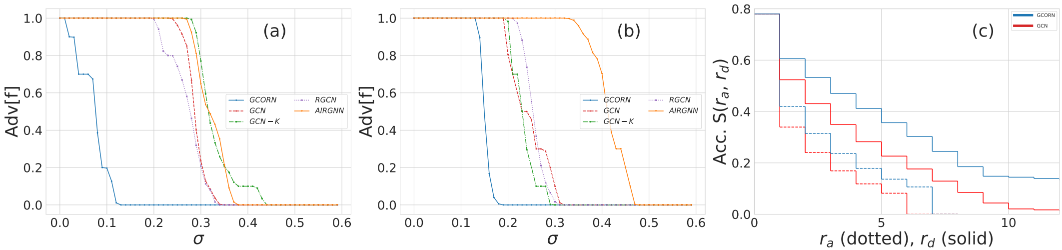
Results - Structural Attacks

Table: Attacked classification accuracy (± standard deviation) of the models on different benchmark node classification datasets after the structural attacks application.

Attack	Dataset	GCN	GCN-Jaccard	RGCN	GNN-SVD	GNN-Guard	ParsevalR	GCORN
Mettack	Cora	73.0 ± 0.7	75.4 ± 1.8	69.2 ± 0.3	73.6 ± 0.9	74.4 ± 0.8	71.9 ± 0.7	$\mathbf{77.3 \pm 0.5}$
	CiteSeer	63.2 ± 0.9	69.5 ± 1.9	68.9 ± 0.6	65.8 ± 0.6	68.8 ± 1.5	68.3 ± 0.8	$\mathbf{73.7 \pm 0.3}$
	PubMed	60.7 ± 0.7	62.9 ± 1.8	65.1 ± 0.4	82.1 ± 0.8	$\mathbf{84.8 \pm 0.3}$	69.5 ± 1.1	71.8 ± 0.4
	CoraML	73.1 ± 0.6	75.4 ± 0.4	77.1 ± 1.1	71.3 ± 1.0	76.5 ± 0.7	76.9 ± 1.3	$\mathbf{79.2 \pm 0.6}$
PGD	Cora	76.7 ± 0.9	78.3 ± 1.1	72.0 ± 0.3	71.6 ± 0.4	75.0 ± 2.0	78.4 ± 1.2	$\mathbf{79.9 \pm 0.4}$
	CiteSeer	67.8 ± 0.8	70.9 ± 1.0	62.2 ± 1.8	60.3 ± 2.4	68.9 ± 2.2	70.6 ± 1.0	$\mathbf{73.1 \pm 0.5}$
	PubMed	75.3 ± 1.6	73.8 ± 1.3	78.6 ± 0.4	81.9 ± 0.4	$\mathbf{84.3 \pm 0.4}$	77.3 ± 0.7	77.4 ± 0.4
	CoraML	76.9 ± 1.2	75.0 ± 2.4	77.5 ± 0.3	73.1 ± 0.5	75.5 ± 0.8	81.3 ± 0.4	$\mathbf{84.1 \pm 0.2}$
DICE	Cora	74.9 ± 0.8	76.9 ± 0.9	79.6 ± 0.3	72.2 ± 1.4	75.6 ± 1.1	$\mathbf{79.7 \pm 0.8}$	78.9 ± 0.4
	CiteSeer	64.1 ± 0.5	66.0 ± 0.6	68.7 ± 0.5	62.6 ± 1.2	65.5 ± 1.1	68.9 ± 0.4	$\mathbf{74.6 \pm 0.4}$
	PubMed	79.4 ± 0.4	78.3 ± 0.2	$\mathbf{79.8 \pm 0.4}$	76.6 ± 0.5	77.8 ± 0.7	79.2 ± 0.3	78.1 ± 0.6
	CoraML	78.3 ± 0.6	77.5 ± 0.3	80.1 ± 0.4	58.7 ± 0.4	77.5 ± 0.2	80.5 ± 1.3	$\mathbf{81.1 \pm 0.8}$

- GCORN is also effective against **structure-based, as well as combined structure and feature attacks**.

Results - Robustness Certificates/Evaluations



(a) and (b) display $Adv_{\epsilon}^{\alpha,\beta}[f]$ for Cora and OGBN-Arxiv. (c) Robustness guarantees on Cora, where r_a, r_d are respectively the maximum number of adversarial additions and deletions.

- Similar performance analysis found using our proposed robustness evaluation and other available certificates.

[1] Efficient robustness certificates for discrete data: Sparsity-aware randomized smoothing for graphs, images and more. Bojchevski *& Al* - ICML 2020.

Is It All Perfect ?

Table: Performance of GCN and our proposed GCORN model, for different used approximation orders, on the Cora dataset.

	GCN	GCORN(1 ord)	GCORN(2 ord)	GCORN(3 ord)
Training Time (in s)	2.8 ± 0.01	4.8 ± 0.07	8.7 ± 0.07	10.9 ± 0.08
Accuracy w/o attack	79.2 ± 1.6	78.8 ± 1.3	79.8 ± 0.9	80.8 ± 1.1
Accuracy w. attack	68.4 ± 1.9	77.1 ± 2.1	78.3 ± 1.1	78.6 ± 0.4

Table: Mean training time analysis (in s) of a our GCORN in comparison to the other benchmarks.

Dataset	GCN	GCN-K	AIRGNN	RGCN	GCORN
Cora	2.8	1.8	2.6	3.2	4.8
CiteSeer	2.4	5.8	2.9	2.4	4.6
PubMed	5.9	8.9	7.4	14.5	7.3
CS	6.1	12.1	12.4	13.8	15.5
Ogbn-Arxiv	77.8	185.8	68.1	161.6	78.4

- Adversarial Robustness is computationally demanding.
- Can we do better ? A method “effective” and “simple”.

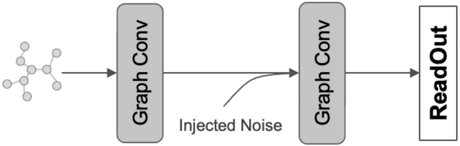
A Simple and Yet Fairly Effective Defense for Graph Neural Networks

Ennadir, Abbahaddou, Lutzeyer, Vazirgiannis & Boström (2024, AAAI)

Problem: Available defense methods suffers from **High complexity and training time** (often increasing with the input graph size).

Solution Approach: We propose a GNN, called the *NoisyGNN*, in which **hidden states are perturbed** by random noise following a normal distribution $N \sim \mathcal{N}(0, \beta I)$, i.e., our GNNs are of the form

$$\hat{y} = \sigma\left(\tilde{A}\,\mathrm{ReLU}\left(\tilde{A}XW^{(1)} + N\right)W^{(2)}\right).$$



Theoretical Results

Theorem (Upper Bounds on GNN Vulnerability).

We consider structural perturbations of the input graph (A, X), with a budget ϵ and 2-layer GNNs with 1-Lipschitz continuous activation functions and weight matrices $W^{(1)}, W^{(2)}$.

- Then, the vulnerability of GCNs is upper bounded by

$$\gamma = \frac{2(\|W^{(2)}\|\|W^{(1)}\|\|X\|\epsilon)^2}{\beta};$$

- Then, the vulnerability of GINs is upper bounded by

$$\gamma = \frac{(\|W^{(2)}\|\|W^{(1)}\|\|X\|\epsilon(2\|A\|+\epsilon))^2}{2\beta}.$$

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$$\gamma = \frac{(\|W^{(2)}\|\|W^{(1)}\|\|X\|\epsilon(2\|A\|+\epsilon))^2}{2\beta}.$$

Insight: Our upper bound on the vulnerability of a GNN is **smaller for large** β yielding a **more robust GNN.**

Experimental Results

Dataset	Attack Budget	GCNGuard	GCN-Jaccard	GCN-SVD	RGNN	NoisyGCN
Cora	Clean	77.5 $\pm$ 0.7	80.9 $\pm$ 0.7	80.6 $\pm$ 0.4	**83.5** $\pm$ **0.3**	83.2 $\pm$ 0.4
	Budget (5%)	75.8 $\pm$ 0.6	78.9 $\pm$ 0.8	78.4 $\pm$ 0.6	78.3 $\pm$ 0.6	**81.2** $\pm$ **0.7**
	Budget (10%)	74.7 $\pm$ 0.4	**76.7** $\pm$ **0.7**	71.5 $\pm$ 0.8	70.7 $\pm$ 0.8	74.5 $\pm$ 0.6
CiteSeer	Clean	70.1 $\pm$ 1.5	71.2 $\pm$ 0.7	70.7 $\pm$ 0.4	**72.3** $\pm$ **0.5**	71.9 $\pm$ 0.4
	Budget (5%)	69.9 $\pm$ 1.1	70.3 $\pm$ 2.3	68.9 $\pm$ 0.7	70.6 $\pm$ 0.7	**72.3** $\pm$ **0.6**
	Budget (10%)	70.0 $\pm$ 1.5	67.5 $\pm$ 2.1	68.8 $\pm$ 0.6	68.7 $\pm$ 1.2	**70.4** $\pm$ **0.8**
PubMed	Clean	84.5 $\pm$ 0.6	85.0 $\pm$ 0.5	82.7 $\pm$ 0.3	**85.1** $\pm$ **0.8**	85.0 $\pm$ 0.6
	Budget (5%)	**84.3** $\pm$ **0.9**	79.6 $\pm$ 0.3	81.3 $\pm$ 0.6	81.1 $\pm$ 0.7	81.8 $\pm$ 0.4
	Budget (10%)	**84.1** $\pm$ **0.3**	67.4 $\pm$ 1.1	81.1 $\pm$ 0.7	65.2 $\pm$ 0.4	73.3 $\pm$ 0.6
PolBlogs	Clean	93.1 $\pm$ 0.6	-	86.5 $\pm$ 0.8	94.9 $\pm$ 0.3	**95.2** $\pm$ **0.4**
	Budget (5%)	72.8 $\pm$ 0.8	-	**85.1** $\pm$ **1.6**	76.0 $\pm$ 0.8	79.7 $\pm$ 0.6
	Budget (10%)	68.7 $\pm$ 1.0	-	**84.8** $\pm$ **2.3**	69.2 $\pm$ 1.2	73.4 $\pm$ 0.5

Table: Node classification accuracy ($\pm$ standard deviation) when subject to Mettack.

- Our NoisyGCNs **sometimes outperform** other defense methods.

Experimental Results - Time Complexity

Table: Mean training time analysis (in s) of the NoisyGNN in comparison to other baselines for both the GCN and GIN instances.

Dataset	GCNGuard	GCN-Jaccard	RGCN	GCN-SVD	NoisyGCN
Cora	28.52	1.93	1.16	1.39	1.29
CiteSeer	36.04	1.58	1.23	1.12	1.24
PubMed	731.26	12.27	34.19	4.60	2.41
PolBlogs	18.17	5.17	0.96	0.80	0.65
Dataset	**GINGuard**	**GIN-Jaccard**	**RGCN**	**GIN-SVD**	**NoisyGIN**
Cora	48.93	3.12	1.31	1.51	1.93
CiteSeer	58.45	3.78	1.44	2.20	2.76
PubMed	963.58	16.28	41.09	6.33	7.86
PolBlogs	43.7	5.52	0.95	3.71	3.16

- NoisyGNNs are **faster to train** than most other defense methods.

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- NoisyGNNs are **faster to train** than most other defense methods.
- When **combined with other defense methods**, best performance is achieved.

Conclusions

- Graph Representation Learning is a highly active area of research at the moment gaining both academic and industrial interest.

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- Graph Representation Learning is a highly active area of research at the moment gaining both academic and industrial interest.
- Graph Neural Networks are a versatile and powerful tool, that you may want to consider using but their adversarial robustness is still subject to questions.

Specifically, with regards to the presented projects:

- Both the introduction of noise and the orthonormalisation of weight matrices are viable avenues towards more robust Graph Neural Networks.

Conclusions

- Graph Representation Learning is a highly active area of research at the moment gaining both academic and industrial interest.
- Graph Neural Networks are a versatile and powerful tool, that you may want to consider using but their adversarial robustness is still subject to questions.

Specifically, with regards to the presented projects:

- Both the introduction of noise and the orthonormalisation of weight matrices are viable avenues towards more robust Graph Neural Networks.
- Aim for the GCORN approach when looking for better adversarial robustness.
- Aim for the NoisyGNN approach when looking for the right trade-off between robustness and time complexity.

Conclusions

- Graph Representation Learning is a highly active area of research at the moment gaining both academic and industrial interest.
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GCORN

NoisyGNN

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